Computations of Shocked Flows on Nonadapted Meshes Using Floating Shock Fitting

Peter-M. Hartwich (ViGYAN, Inc.)

Transonic Aerodynamics Branch
Applied Aerodynamics Division

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Outline

- Governing Equations
- Numerical Method
- Results
- Summary
Governing Equations

\[ Q_t + AQ_\xi + BQ_\eta = 0 \]

\[ Q = (a, u, v, s)^T \]

\[
A = \begin{pmatrix}
V & \xi_x a \delta & \xi_y a \delta & 0 \\
\xi_x a / \delta & V & 0 & -\xi_x a^2 \\
\xi_y a / \delta & 0 & V & -\xi_y a^2 \\
0 & 0 & 0 & V
\end{pmatrix}
\]

where

\[ V = \xi_x u + \xi_y v \]
Time Differencing

\[
\begin{align*}
[I - \psi & \tau (A_{i+1/2}^- \Delta i+1/2 + A_{i-1/2}^+ \Delta i-1/2) \\
& - B_{j+1/2}^- \Delta j+1/2 + B_{j-1/2}^+ \Delta j-1/2)]^n \Delta Q^n \\
& = -\tau \cdot RES(Q^n)
\end{align*}
\]

- Euler backward (1st order)
- Crank-Nicolson (2nd-order)
- AF algorithm
  - diagonalized version
  - tridiagonal inversion
Split Coefficient Matrix (SCM) Method

\[ Q_t - A^- \Delta_{i+1/2}^+ Q + A^+ \Delta_{i-1/2}^- Q - B^- \Delta_{j+1/2}^+ Q + B^+ \Delta_{j-1/2}^- Q = 0 \]

with

\[ \Delta_{i+1/2}^+ Q = (3/2 - \phi_1) \Delta_{i+1/2} Q - (1/2 - \phi_1) \Delta_{i+3/2} Q \]

\[ \Delta_{i-1/2}^- Q = (3/2 - \phi_2) \Delta_{i-1/2} Q - (1/2 - \phi_2) \Delta_{i-3/2} Q \]

shock between \( i \) and \( i - 1 \):
Two-Dimensional Shocks

- shock orientation:

\[ N = \frac{\Delta q}{|\Delta q|} \]

\[ q = \begin{pmatrix} V \\ V^* \end{pmatrix} = \begin{pmatrix} \hat{\eta}_x u + \hat{\eta}_y v \\ \hat{\eta}_y u - \hat{\eta}_x v \end{pmatrix} \]

- shock detection:

\[ \Sigma = \frac{(a_B - \delta \Delta_{i-1/2} U)}{a_A} \]

\[ U = \hat{\xi}_x u + \hat{\xi}_y v \]
Organization of Shock Points

- based on one of Moretti's more recent techniques:
  - shock point information is stored in 1-D arrays
    - shock Mach number $M(\Sigma)$, $\Sigma(J)$
    - relative shock location $SPL(J)$
    - marker for high-pressure side $IHP(J)$
  - absolute shock location is defined by $IJ(i_J, j_J) = J$
Solutions for Riemann's Problem

- present calculation
- Steger & Warming
- exact solution

solution at time = 1

Initial pressure ratio = 10
Pressure Distributions for Shock Reflection Problem

\[(C_p)_{\text{max}} = 1.0\]
\[(C_p)_{\text{min}} = 0.0\]
\[\Delta C_p = 0.05\]
Pressure Coefficients for Shock Reflection Problem

16x6 grid points
$y/l = .40$

31x11 grid points
$y/l = .40$

61x21 grid points
$y/l = .40$
Convergence Summary for Shock Reflection Problem

local time stepping, CFL = 10

grid
- 16x6
- 31x11
- 61x21
Pressure Field for Bow Shock Problem

\((C_p)_{\text{max}} = 2.0\)
\((C_p)_{\text{min}} = 0.0\)
\(\Delta C_p = 0.1\)

21x11 grid points

\((C_p)_{\text{max}} = 2.0\)
\((C_p)_{\text{min}} = 0.0\)
\(\Delta C_p = 0.1\)

41x21 grid points
Mach Number Distribution for Bow Shock Problem

\[ M_{\text{max}} = 8.0 \]
\[ M_{\text{min}} = 0.0 \]
\[ \Delta M = 0.1 \]

21x11 grid points

\( \Delta \) shock shape after Lyubimov & Rusanov

41x21 grid points

\( \Delta \) shock shape after Lyubimov & Rusanov
Surface Pressure Distribution for Bow Shock Problem

21x11 grid

41x21 grid

$C_p$ vs $\theta$, circumferential angle, deg

- present calculation
- Lyubimov & Rusanov
Accomplishments

- first time-implicit floating shock fitting scheme
- computational efficiency is comparable or better than that of existing time-implicit shock-capturing schemes
- shocks are accurately computed on crude and unadapted meshes

Future Work

- transonic airfoil calculations
- floating contact fitting technique
- extension to 3-D