ON THE INFLUENCE OF THE GEOMETRY OF SLENDER BODIES OF REVOLUTION AND DELTA WINGS ON THEIR DRAG AND PRESSURE DISTRIBUTION AT TRANSONIC SPEEDS

by

F KEUNE and K OSWATITSCH

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Friedrich Keune and Klaus Oswatitsch

SUMMARY

With the help of the area rule the flow around delta wings and swallow-tailed wings in the transonic range is referred to the flow around (equivalent) bodies of revolution. Since the critical Mach-numbers - the limits of the transonic range and the linearizable subsonic and supersonic region - lie quite near unity, the transonic properties can be determined by the calculation of the critical Mach-numbers, the calculation in the linearized ranges, and at Mach-number 1. For the latter the "parabolic method" /6/ is used and applied together with a transonic method of characteristics to various bodies. The lower critical Mach-number is determined with sufficient accuracy by the linear theory. For the upper critical Mach-number the formula of Oswatitsch-Sjödin /15/ is used.

After general considerations a geometrical system of bodies of revolution is introduced in the second section. The suitability of this system is checked by applying to the linear range in section 3. Section 4 gives some examples for wings equivalent to the bodies of revolution considered. In the next section the sonic flow is calculated and in the last section the results are given in a most general form by the transonic similarity.

The drag of the forebody (body before the maximum thickness) at $M_\infty = 1$ is, according to /6/, half the linearized supersonic drag. This report shows now that the drag of the afterbody is practically equal to the whole supersonic drag of this part. Therefore the drag of the whole body at $M_\infty = 1$ lies nearer the supersonic value the greater the contribution of the afterbody is, or the shorter this part is.

It was known to the authors /5/ that the linear subsonic theory holds very well for all subcritical Mach-numbers $M_\infty$. The calculations made in connection with the present work show that even the linear supersonic theory holds for all supersonic Mach-numbers. The small subsonic region on the tip at the uppercritical Mach-number has no appreciable influence.

The area rule requires a correction if the wings or the equivalent bodies of revolution have too blunt ends. This is not a very serious problem, because in the important Mach-number range the ends of those wings lie in supersonic flow. This matter will be treated later.

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1. THEORETICAL BACKGROUND.

In accordance with the area rule of K. Oswatitsch, which is the law of equivalence for low-aspect-ratio wings in transonic flow (cf. K. Oswatitsch /1/ and K. Oswatitsch - F. Keune /2/), the flow around wings is referred to the flow around bodies of revolution. In the present paper, several bodies of revolution are considered, and the equivalent wings for some of these bodies determined. The thickness distribution for these wings is fixed for given plan forms - the delta wing and swallow-tail wing - by the agreement of the cross-section at all points on the chord with that of the body of revolution. The flow on the body of revolution and the wing at each point on the chord then differs only by the difference in the cross-sectional flows. For the body of revolution, the flow at each cross section is that of a single source with the value on the body obtained from Eq. (12), and for the wing, it is that of a source distribution ( /2/, Eq.(12)). It is invariable for all Mach numbers, and it conforms to the boundary conditions for the flow on the surface of the body. For equivalent bodies, the difference between u-components (in the direction of the axis of the body) and pertinent cross-sectional flow is the same. It is called the spatial influence, and it includes the total Mach number influence in the non-linear and linear theory.

Laws of similarity for transonic velocities are given in another work (K. Oswatitsch - F.Keune /3/), and the drag for low-aspect-ratio wings at zero lift is obtained from a generalization by G.N. Ward /4/; the results are used in the calculations made here.

At the beginning of Section 6 it is demonstrated that in calculating the flow on bodies of revolution, the results of the non-linear and linear theories are in practical agreement at the Mach number of shock wave separation. In subsonic flow there is a difference of about 6% between the maximum velocities at the sub-critical Mach number of the spindle (see F. Keune - K. Oswatitsch /5/, Fig. 18), but the sub-critical Mach number itself can be very well approximated through the linear theory.

The flow on bodies of revolution at sonic velocity and zero lift has not been dealt with in any of the papers mentioned as yet. In a work by K. Oswatitsch and F. Keune /6/, based on a proposal by Oswatitsch, this gap was filled with respect to half-bodies. The project was described by Oswatitsch at the Brooklyn Conference in New York in late January. The major problem - that of the transition from subsonic to supersonic flow on bodies of revolution - was solved here. The application of this theory to an entire body of revolution of finite length, the calculation of the lower and upper critical Mach numbers, and the comparison of the results with supersonic values constitute the main part of the present paper.

For the linear theory, papers by F. Keune /7/ and F. Keune and K. Oswatitsch /8/ are the principal sources. Papers by M.C. Adams and W.R. Sears /9/ and by M.A. Heaslet and H. Lomax /10/ lead to the same results. The substitution of the boundary conditions and the calculations on the surface of the body were done with the usual approximations. More accurate results can be obtained without very much additional work (see ref. /11/, /12/).

In one case, the Sauer-Heinz method of linear characteristics (cf. K. Oswatitsch /13/, p.317)
was applied. The results were compared with Oswatitsch's method of non-linear characteristics for transonic flow /14/; very little difference was found.

2. GEOMETRY OF THE BODIES OF REVOLUTION

It was found advantageous to construct the bodies of revolution of two half-bodies which are essentially independent of each other. At the point of maximum thickness, the forebody and afterbody meet in such a way, that at these points \( x = x_0 \) on the axis of the body, not only the cross-sectional area \( Q(x) = Q(x_0) \), but also at least the first two derivatives \( Q_x(x_0) \) and \( Q_{xx}(x_0) \) should agree. If \( \Upsilon \) is the ratio of the maximum radius \( \Upsilon x_0 \) to the chord of the forebody \( x_0 \), then the maximum cross section \( Q_{\text{max}} = \Upsilon \pi x_0^2 \) (see Fig. 1), and in addition \( Q_x(x_0) = 0 \). With the maximum cross section as the geometrical parameter of similarity of the bodies of revolution and the wings, the characteristic parameter shape of the forebody is found by

\[
-12 = \frac{Q_{xx}(x_0)}{\pi \Upsilon^2} = 0
\]

For the body selected here, this parameter lies within the limits given in (1). The extent of the forebody is

\[
0 = \frac{x}{x_0} \leq 1
\]

and that of the afterbody

\[
1 = \frac{x}{x_0} = \frac{L}{x_0} \quad \text{or} \quad 1 = \lambda \leq 2
\]

with the dimensionless coordinate

\[
\lambda = \frac{L - 2x_0 + x}{L - x_0}
\]

and the body length \( L \). The ratio of the length of the forebody to that of the afterbody is useful as a specific measure of the location of maximum thickness:

\[
B = \frac{L - x_0}{x_0}
\]

the "ratio of elongation" of the afterbody; thus, for long afterbodies \( B > 1 \), and for short afterbodies \( B < 1 \).

The forebody must be pointed if the theory is to be applicable. However, the geometrical system we are applying here allows us to select a constant \( b_0 = 1 \) for pointed afterbodies and \( b_0 = 0 \) for rounded afterbodies. In addition, two other constants \( b_2 \) and \( b_3 \) are assumed for the afterbody.
The analytical forms of both bodies are given in Appendix I.

With the assumptions regarding pointed afterbodies \((b_1 = 1)\), which apply throughout the following, and with \(b_2 = b_3 = 0\) the afterbody as well as the forebody is monoparametrically dependant on parameter \((1)\) alone. However, the parameter which is geometrically possible for the afterbody with a ratio of elongation of \(B < 1\) then lies with considerably narrower limits than could be given in \((1)\) for the forebody. In such cases, the parameters \(b_2 = 0\) and \(b_3 = 0\) are important. A necessary (though not by itself adequate) condition for the existence of the afterbody is that the second derivative of the cross-sectional area at the trailing edge, \(\frac{1}{\pi t^2} \cdot Q_{xx}(L) > 0\), be greater than zero (see Fig. 2). With Appendix I, a short calculation \((\text{Eq. (6) to (10)})\) yields the following results:

\[
b_2 = -\frac{1}{2} \cdot \frac{Q_{xx}(x_b^*)}{\pi t^2} B^2 - 6
\]

On the other hand, for the given cross-sectional area at the trailing edge, \(\frac{1}{\pi t^2} Q_{xx}(L) > 0\), we obtain the following value for parameter \(b_2\):

\[
b_2 = \frac{1}{2} B^2 \left( \frac{Q_{xx}(L)}{\pi t^2} - \frac{Q_{xx}(x_b^*)}{\pi t^2} \right) - 6
\]

In both formulae, \(\frac{1}{\pi t^2} Q_{xx}(x_b^*) < 0\) is always less than zero, so that the first term on the right side is occasionally positive.

As a result of this method of determining \(b_2\), parameter \(b_3\) is not affected. It is suitably determined by the geometric form of the body at location \(x_{x_b}^* = 1\) or \(X = 1\) at the junction of the forebody and afterbody. This establishes how the third derivative \(\frac{1}{\pi t^2} \cdot \frac{Q_{xxx}(x_b^*)}{\pi t^2}\) of the afterbody will deviate from that of the forebody. If these derivatives for the two bodies do not agree, there will be a discontinuity (as in \(Q_{xx}(x)\) - see Fig. 2) not only for the cross-sectional flow, but also for the form of the velocity distribution in the transonic similarity, at this point \(x = x_b^*\). For the forebody \((\text{cf. Appendix I}), we obtain:

\[
\frac{1}{6} \left( \frac{Q_{xxxx}(x_b^*)}{\pi t^2} \right)_{\text{forebody}} = 4 + \frac{Q_{xx}(x_b^*)}{\pi t^2}
\]

and for the afterbody:

\[
\frac{B^3}{6} \left( \frac{Q_{xxxx}(x_b^*)}{\pi t^2} \right)_{\text{afterbody}} = -4 - B^2 \frac{Q_{xx}(x_b^*)}{\pi t^2} + (b_2 + b_3)
\]

From this follows, as a parameter characteristic for the flow,

\[
b_2 + b_3 = -\frac{B^3}{6} \left( \frac{Q_{xxxx}(x_b^*)}{\pi t^2} \right)_{\text{afterbody}} - \left( \frac{Q_{xxxx}(x_b^*)}{\pi t^2} \right)_{\text{forebody}} + 4(1 - B^2 - B^2 \frac{Q_{xx}(x_b^*)}{\pi t^2})
\]
In the special case where the third derivatives \( \frac{1}{\pi t^2} x_0^3 Q_{xxx}(x_0) \) of both bodies agree and where the value of the spindle \( \frac{1}{\pi t^2} Q_{xx}(x_0) = -4 \) one gets for various locations of maximum thickness:

\[
\frac{Q_{xx}(x_0)}{\pi t^2} = -4 \quad b_2 + b_3 = 4(1 - B^2)
\]

\( x_0 = 0.35 \quad = -9.795 \)

\( x_0 = 0.50 \quad = 0 \)

\( x_0 = 0.65 \quad = 2.84 \)

Since \( \frac{1}{\pi t^2} Q_{xx}(x_0) \) must always be less than zero, the sum of \( b_2 \) and \( b_3 \), when the third derivatives agree, will be positive where \( B \) is less than 1 and negative where \( B \) is greater than 1. When \( B = 1 \) and \( x_0 = \frac{1}{2} L \), \( b_2 + b_3 \) is independent of \( \frac{1}{\pi t^2} Q_{xx}(x_0) \), and with the application of Eq. (11) it becomes zero.

With \( b_0 = 0 \), other relationships for \( b_2 \) and \( b_3 \) can be obtained with Eq. (6) to (11).

The linear theory (/7/ and /8/) leads directly to the basis for a significant choice of the parameter of shape.


The system chosen results in bodies which at point \( x = x_0 \) are not continuous in all derivatives of the cross-sectional area \( Q(x) \). Therefore the equations of the linear theory for the flow given in F. Keune /7/ and the formula for the linear drag given in K. Oswatitsch - F. Keune /3/ are sometimes to be integrated only over the intervals \( 0 \leq x_0 \leq 1 \) and \( 1 \leq x \leq 2 \). If we divide the component of the perturbation flow in the direction of undisturbed flow \( \frac{U}{\pi t^2} \) into the cross-sectional flow over bodies of revolution

\[
\frac{U}{\pi t^2} = \frac{\Phi_x(x, \vec{y} = \vec{h}(x))}{\pi t^2} = \frac{1}{2\pi} \frac{Q_{xx}(x)}{\pi t^2} \ln \frac{\vec{h}(x)}{x_0}
\]

the spatial influence \( \Omega_x(x) \pi t^2 \) at \( M_\infty = 0 \) or \( M_\infty = \sqrt{2} \), and the Mach number influence of the linear theory, then the component can be obtained from

\[
\frac{U}{\pi t^2} = \frac{\Phi_x(x, \vec{h}(x))}{\pi t^2} + \frac{\Omega_x(x)}{\pi t^2} + \frac{1}{2\pi} \frac{Q_{xx}(x)}{\pi t^2} \ln \sqrt{M_\infty^2 - 1}
\]

The calculation of the spatial influence by the use of /7/ or /8/ leads to the formulae given in Appendix II for forebodies and afterbodies in the two intervals to be differentiated, and for subsonic velocities. Tables are provided in the appendix for the numerical calculation. In addition, the distance
influence of the forebody upon the flow over the afterbody, and also, for \( M_\infty < 1 \), that of the afterbody on the flow over the forebody are given in diagrams. These distance influences are dependent on the given location of maximum thickness \( x_*/L \) or \( B \), since it is this which determines the distance of a point on one of the bodies from that on the other. Thus Appendix II allows calculating the velocity distribution in accordance with the linear theory for all bodies of revolution in the system.

The geometrical magnitudes \( Q_{xx}(x_*)/\pi L^2 \) and \( B(x)/x_0^2 = \sqrt{Q(x)/\pi L^2} \) required for the cross-sectional flow and the linear Mach number influence are available directly from Appendix I.

For the contribution of pressure to the drag of the body of revolution, the drag coefficient

\[
C_D = \frac{D}{\frac{1}{2} \rho_\infty U_\infty^2 Q_{\max}} = \frac{D}{\frac{1}{2} \rho_\infty U_\infty^2 \pi \tau_f x_0^2}
\]

is calculated in the linear theory from \(/3/\) and at sonic velocity from \(/6/\).

At sonic velocity, for the forebody we obtain with \(/6/\):

\[
M_\infty = 1
\]

\[
\frac{C_D}{\tau_f} = 3 + \frac{1}{4} \frac{Q_{xx}(x_0)}{\pi \tau_f^2} + \frac{1}{4} \left( \frac{Q_{xx}(x_0)^2}{\pi \tau_f^2} \right)^2
\]

The drag of the half-body for the geometrical forms chosen here has, for \( Q_{xx}(x_0)/\pi L^2 = -6 \), a minimum:

\[
C_D = \frac{2}{4} \tau_f^2
\]

For the entire body, the drag at sonic velocity is obtained by numerical integration from the results for the velocity distribution (see Sect.5).

In supersonic flow, the drag \(/14/\) according to the linear theory \(/3/\) is composed of three components: the forebody component \( (C_D)_{\tau_f} \), the afterbody component \( (C_D)_{\sqrt{2}} \), and a small additional drag, originating in the influence of the forebody on the flow over the afterbody (see Appendix III).

The part of the drag caused by the forebody is, according to \(/6/\), exactly twice as great as at sonic velocity:

\[
(C_D)_{\tau_f} = 2(C_D)_{\sqrt{2} M_\infty = 1}
\]

For the given bodies, the additional drag \( (C_D)_{\sqrt{2}} \) is one order of magnitude smaller than \( (C_D)_{\tau_f} \) is and it can be obtained by a numerical integration in accordance with the Simpson rule. Drag component \( (C_D)_{\sqrt{2}} \) can be calculated analytically in the linear theory. The results are given in Appendix III and Fig. 3.
In Fig. 3, the drag coefficients are plotted as functions of the thickness ratio \( T \) of the forebody, and also as functions of the thickness ratio \( \delta \) of the entire body:

\[
\delta = \frac{2x_5}{L} \tag{17}
\]

In addition, the limits of the geometrically possible body shapes, which can easily be found, are also given here. Fig. 3 a) gives the values for the forebody alone \((M_\infty = \sqrt{2})\), while b) gives them for the entire body with the parameter \( b_2 = b_3 = 0 \), and in c) and d) the parameter \( q_{xx}(x_0)/\pi t^2 = -4 \) is used.

All the curves are parabolas with more or less pronounced minima. \( q_{xx}(x_0)/\pi t^2 = -4 \), which was chosen for definite reasons (see Sect. 4) as the parameter for the body shapes, is advantageous for locations of maximum thickness \( x_0/L < 1/2 \), while for large locations of maximum thickness \( x_0/L > 1/2 \), it gives very large drag coefficients.

This fact is quite understandable. The parameter is a measure of the radius of curvature of the body at the point of maximum thickness. For large locations of maximum thickness, it is too large to allow the afterbody to terminate within a short distance without coercive measures (very large \( q_{xx}(x) \); see Fig. 2). With the normal component \( v \) of the velocity, \( v_y \sim q_{xx}(x) \) on the body (see also Fig. 1). The integral of \( v_y \) from \( x=0 \) to \( x=x_0 \) must be equal to the integral of the extension \( x \leq x \leq L \) of the afterbody. For the relatively slight inclination \( q_{xx}(x) = -4 \pi^2 t^2 \), the normal component on the body at large locations of maximum thickness must increase greatly to meet the other requirements. However, this great increase in the normal velocity has unfavorable effects on the velocity distribution and the drag.

Thus, without changing the drag of the forebody, considerably more favorable drag values can be obtained for \( q_{xx}(x_0)/\pi t^2 = -8 \) with a location of maximum thickness \( x_0/L = 0.65 \) than those which appear here. The monoparametric form itself shows better results in Fig. 3 b.

For the location of maximum thickness \( x_0/L = 1/2 \), the parameter \( q_{xx}(x_0)/\pi t^2 = -6 \) appears to be the most favorable, for the entire body as well as the halfbody. However, determination of the minimum drag is not the purpose of this report.

4. THE CHOOSEN BODIES OF REVOLUTION AND THE EQUIVALENT DELTA WINGS

The half spindle was employed for calculating the flow at sonic velocity over forebodies 6/, as numerical values were available for it. When 6/ is applied, the results obtained with this theory agree very well with the numerical values, and therefore it appears worthwhile to use them for the calculation of the flow over the entire body.

However, this establishes the half spindle here as the forebody for all bodies of revolution. For this forebody, \( q_{xx}(x_0)/\pi t^2 = -4 \). As afterbody the entire spindle, \( x_0 = 1/2 L \), is the first of the chosen bodies. Fig. 3 does not indicate that any great improvement in the drag can be expected with the same location of maximum thickness \( x_0 = 1/2 L \), and therefore we select as Body II an afterbody which does not differ greatly from the one first chosen. Fig. 3 shows that when the point of maximum thickness
is near the nose $x_5 = 0.35 L$ there is no body with the parameter $b_2 = b_3 = 0$; we therefore select - as Body III (Fig. 34) without the third derivative $Q_{xxx}(x_5)$ in agreement, and Body IV (Fig. 3c) with the third derivative in agreement - two bodies with drags $C_D/\tau^2$ which will differ at supersonic velocity by about 10%. For large locations of maximum thickness $x_6 = 0.65 L$, the drag differs greatly as opposed to the two other locations of maximum thickness from Fig. 3, depending on the choice of the shape parameter $b_2$ and $b_3$. Body V is taken from the monoparametric curves, Fig. 3b. The two other bodies, VI and VII, were chosen from Fig. 3 at equal locations of maximum thickness to obtain a great difference in drag (readable in Fig. 3 only for $C_D/\delta^2$).

The form of the body of revolution and the first two derivatives $Q_x(x)$ and $Q_{xx}(x)$, which have an important influence on the flow, are given in Fig. 1 and Fig. 2. Table 1 contains the shape-parameters of the chosen bodies and their drag coefficients at $M_\infty = \sqrt{2}$. The coefficients also included for sonic velocity will be discussed later.

Body IV in particular has an unusual shape for bodies of revolution, but it can very well be used for any strongly swept wing shape.

Bodies I, III and V are used in dealing with the low-aspect-ratio wing. For body I and body V we select the delta shape as the equivalent wing, and for body I and body III the swallow-tail shape. The choice of equivalent wings for the given bodies of revolution is very plentiful, as can be seen from /3/ Fig. 3.

In any case, in accordance with the assumptions of the theory set forth in Section 1, the local span at the leading edge of the wing must disappear. In addition, the wing tip in the present case must be assumed to be linear from the leading edge to the maximum span at the trailing edge.

Under these conditions, the maximum thickness of all wings occurs at point $x = \frac{2}{3} x_5$ of the wing root ($x$-axis), since all equivalent bodies of revolution have the same distribution of the cross-sectional areas and $Q_{xx}(x_5)/\tau^2 = -4$ between $0 \leq x \leq x_5$.

With the maximum span $6 L$ the local span at the maximum cross-section will then always be $x_6 \delta/L$, and the maximum cross-section for bodies of revolution and wings will be

$$Q_{max} = \pi \tau^2 x_6^2 = 4 \psi(x) h(x_6; 0) x_6 \delta$$  \hspace{1cm} (18)

where $h(x/L, 0)$ is the thickness distribution at the wing root and $\psi(x)$ is the area factor of the cross-sectional area of the wing. For elliptical, parabolic, or rhombic cross-sections, this area factor is $\pi/4$, $\sqrt{3}/4$, $1/2$. Naturally, $\psi(x)$ as well as $h(x/L, 0)$ can be a function of $x$. With a local span of $b(x)$, the thickness distribution $h(x/L, 0)$ or $\psi(x)$ is then determined from the cross-sectional area of the body of revolution:

$$Q(x) = 4 \psi(x) h(x; 0) b(x)$$  \hspace{1cm} (19)

For the sonic law of similarity, the thickness distribution $h(x/L, 0)$ and the area factor are of secondary importance. Only the ratio of the maximum span to the chord $\delta$, and the ratio of the maximum
thickness to the chord $\delta_{\text{wing}}$ of the delta wing

$$\delta_{\text{wing}} = \frac{x_0}{L} \left( \frac{x_0}{L} \right)^{1/2}$$

have particular importance. The $\delta_{\text{wing}}$ changes from case to case, although the maximum cross-section remains the same.

The equivalent delta wings for bodies I and V are shown in Fig. 4 for a parabolic cross-section, $\wp(x) = 2/3$.

For swallow-tail wings, the thickness ratio is formed to suit the local chord $t_0$ at the wing root, so that we write $t_0$ in Eq. (20) instead of L. In addition, we make two other assumptions for these plan forms:

a) The maximum wing thickness for all longitudinal sections is located on a straight line from the maximum thickness at $x = \frac{2}{3} x_0, z = 0$ to the maximum span $x = L, z = L/2$ (see Fig. 5).

b) In cross-section, the wing shape remains parabolic, with its maximum located on the line fixed in a).

These requirements determine $t_0$. Fig. 5 shows the swallow-tail wings corresponding to bodies I and III. At point $x = t_0, z = 0$, the root of the trailing edge, certain difficulties may arise in calculating the flow. Details of question are dealt with in /2/, p. 38, and Eq. (59). The flow on a swallow-tail wing is also calculated there.

By changing the area factor $\wp(x)$, which is $\wp = 2/3$ in the given example, the distribution of the local thickness maximum in the cross-section can be varied as desired. The thickness distribution selected affects only the velocity distribution on the wing and the subcritical Mach number, not the drag nor the transonic similarity.

5. THE FLOW AT MACH NUMBER ONE

The flow on a pointed body of revolution at sonic velocity (Fig. 6) begins at the front with pure subsonic flow back to the sonic velocity line, which extends outwards from the body into space, inclining downstream. Behind this line, pure supersonic flow prevails back to the vicinity of the tail, where the supersonic flow generally changes to subsonic flow through a slight shock wave beginning at a certain distance from the body. Thus, while the subsonic and supersonic flows at sonic velocity ($M_\infty = 1$) from regions limited by the sonic velocity line and shock wave, which extend far out into space, the supersonic region in supercritical subsonic flow ($M_\infty < 1$) and the subsonic region in supersonic flow ($M_\infty > 1$) contract to finite values (like a cushion - see Fig. 5, ref. /6/), extending from the body out to finite distances of greater or lesser magnitude in the main flow.

At sonic velocity, the parabolic gas-dynamic equation of K. Oswatitsch and F. Keune /6/ is used to calculate the subsonic flow on the point of the body and the sonic velocity line. In the adjacent supersonic region, the flow is constructed with the method of non-linear characteristics outlined.
in /6/, Sect. 4. When the supersonic flow decreases to sonic speed, intersections of the Mach lines at a certain distance from the body are obtained for all the given bodies (e.g., Fig. 6); these intersections result in shock waves.

For this interval—which with the exceptions of bodies III and IV is limited to the tail of the body—theoretically simplified assumptions were made for the analytical calculation of the subsonic flow on the body (see Appendix IV). The results thus obtained cannot vary greatly from the actual conditions; the drag coefficients obtained for body III are probably somewhat too large (see Table 1).

A special difficulty, not dealt with here, arises with body IV (Fig. 1) at sonic velocity. The transition from supersonic to subsonic velocity on the body occurs at about \( x = 1.65 x_0 \). However, with this body a return to supersonic velocity is to be expected in the neighborhood of \( x = 2.3 x_0 \) before the flow at the tail passes again to subsonic velocity. This presents a problem, as yet unsolved, for bodies of revolution of the shapes possible as equivalent bodies for the given wings /2/.

For the numerical calculation of the drag coefficient (14) of the bodies of revolution, the results given in ref. /3/ are useful. For a vanishing derivative of the cross-sectional area at the trailing edge (see /3/, Eq. (37)), the drag is obtained from the spatial influence of the flow alone. The same applies (Eq. (39), ref. /3/) for equivalent wings. Both the other components of the velocity on the body—the cross-sectional flow and the additive term obtained from the normal component—are exactly zero in the drag integral under the given conditions. Eq. (13) then becomes available for the total drag of the body:

\[
C_D = \frac{2}{Q_{\text{max}}} \int_0^L \frac{W-U_\infty}{U_\infty} Q_x(x) \, dx = -2\pi \int_0^{L/6} \frac{2\pi}{\tau^2} \frac{Q_x(x) Q_y(x)}{\pi \tau^2 x_\delta} \, dx
\]

Table 1 shows the accuracy for body V of this procedure, which is numerically a considerable simplification. For components of the total drag up to a chosen point \( x = x_0 \) on the axis of the body, the equations from ref. /3/ mentioned above are used with the same simplifications of the integration and the addition of only one of the terms dependent on the local cross-sectional area.

6. RESULTS OF CALCULATIONS AND TRANSONIC SIMILARITY.

The accuracy of the linear theory /7/ and /8/ for Mach numbers slightly greater than that for shock wave separation has been checked with the aid of the method of characteristics. For the spindle (body I) and the thickness ratio \( \delta = \infty = 0.146 \) at \( M = 1.197 \) or \( \mu = 2.015 \) (see eq. (22)), the non-linear method of K. Osmatitsch /14/ and the linear method of Sauer-Heinz (see /13/, p. 317) gave practically complete agreement for the pressure distribution \( c_p \). The overlapping of the curves indicates that the drag would show still better agreement. Equally good agreement was found for the Mach number of shock wave separation /14/. The applicability of the linear theory throughout the
subsonic range has been discussed in chapter 1. Thus calculations with non-linear theories need only be made in the range between the lower \( \text{M}_{c1} \) and upper \( \text{M}_{c2} \) critical Mach numbers. The perturbation velocities \( \frac{u}{\pi \text{T}^2} \) for subsonic and supersonic flow for all bodies according to the linear theory are therefore given in table 2, and only those at \( \text{M}_\infty = 1 \) (or \( \mu = 0 \)) are derived from the non-linear theory according to chapter 5. The same applies to the drag coefficients in table 1 and all subsequent results of this chapter.

The pressure distributions for the three main bodies I, III and V at sonic speed are shown in fig. 8. Since the forebodies are the same for constant \( \Upsilon \), here \( \Upsilon = 0.146 \), the pressure distributions differ only on the afterbodies, which have different lengths \( (L = 2, L = 2.857 \text{ and } L = 1.539) \).

All further results are given with the effects of transonic similarity included, see ref. /3/. With \( \text{M}_\infty^* = \frac{\text{M}_\infty}{c*} \) (\( c^* \) is the critical velocity) and thickness ratio \( \Upsilon \) of the forebody or halfbody, the transonic Mach parameter \( \mu \) is introduced:

\[
\mu = \frac{1}{\pi \text{T}^2} \left( 1 - \frac{1}{\text{M}_\infty^*} \right)
\]  

(22)

The choice of this thickness ratio \( \Upsilon \) is particularly favourable for the chosen geometric system, since the results for the forebody at \( \text{M}_\infty = 1 \) will then be the same regardless of the length of the body. In the usual conception, reference to the thickness ratio \( \delta \) of the whole body makes for greater clarity.

The reduced drag coefficient \( \frac{c_D}{\pi \text{T}^2} \) of transonic similarity for bodies of revolution /5/ eq. (46) is therefore plotted in fig. 9 against \( \frac{1}{\pi \text{T}^2} \left( 1 - \frac{1}{\text{M}_\infty^*} \right) \) (see eq. (17)) outside the half-body. The sub-critical Mach number and the sonic and (linear) supersonic drag coefficients are shown, together with the super-critical Mach number. The curves joining these values are shown broken, since they are not fixed completely by further points.

The lower critical Mach number \( \text{M}_{c1} \) defined by the first occurrence of sonic velocity on the surface of the body, does not obey the transonic similarity rule. Only the spatial influence does obey it, but not the perturbation velocities, which also include the logarithmic term of the cross-sectional flow as a term of the sum (see also eq. (26)). This is not the case with the upper critical Mach number \( \text{M}_{c2} \), which is defined by the bow wave separating from the nose of the body. If, however, the upper critical Mach number were defined analogous with the lower by the first occurrence of sonic velocity in the supersonic flow on the body, then transonic similarity would also be lost in this case, as is apparent from the work of K. Oelwasser and L. Sjödin /15/.

The lower critical Mach numbers \( \text{M}_{c1} \) for thicknesses \( \delta = 0,06, 0,12 \text{ and } 0,18 \) are shown in fig. 9. The value for small \( \delta \) lies at lower values of \( \mu \) but naturally closer to \( \text{M}_\infty = \text{M}_\infty^* = 1 \). The dependence of \( \text{M}_{c1} \) on \( \delta \) would still not mean any dependence of the drag rise on \( \delta \) in fig. 9, since, as is well known, this rise always becomes apparent a little after \( \text{M}_{c1} \) is exceeded. The rise is caused by the extension of the supersonic region over the afterbody. Now, on bodies of revolution this region terminates in a very steep fall in velocity, and transonic similarity is always subject to errors at such a point. Thus the similarity is also impaired in the region of the first steep drag rise. Separate curves are obtained for various values of \( \delta \). What applies to bodies of revolution of different thickness...
also applies of course to the conversion to equivalent wings: it is inaccurate in this range.

As is shown by the linear theory of supersonic and transonic flow according to fig. 8, the flow at the conical end of a body of revolution changes abruptly to \( M < 1 \), and therefore transonic similarity will be impaired within certain limits. However, the drag contributions have no longer any appreciable effect there, so that between \( M_\infty = 1 \) and the upper critical Mach number there is essentially only one single curve satisfying transonic similarity.

The tangent at the point \( M_\infty = M^* = 1 \) according to H.W. Liepmann - A.E. Bryson Jr. /16/ gives good check points for the halfbody. In our notation

\[
M_\infty = 1 : \quad \frac{d}{d\mu} \left( \frac{C_D}{1/\pi} \right) = 2
\]

The reduced drag coefficient for the afterbody in the case of bodies I, III and V is found to be the same at sonic speed as at supersonic speed (linear). The result is understandable, since at \( M_\infty = 1 \) the flow on the afterbody has already all the characteristics of flow in supersonic main stream. It is thus possible to express the following rule of thumb for \( C_D \) in transonic flow:

The value of \( C_D \) at \( M_\infty = 1 \) is equal to half the supersonic drag of the forebody (according to the linear theory) plus the full value of the supersonic drag of the afterbody.

It follows that the tangent for the complete body is also given by eq. (23). Certainly, this does not appear to hold so well for body V, but because of the heavy curvature of the \( C_D \) curve at \( \mu = 0 \) the tangent is of no importance.

The above-mentioned rule of thumb is also valid for bodies II and IV, as may be seen from table 1. There are discrepancies in the case of bodies VI and VII, but these two bodies with their rapidly tapering tails call for a critical examination of the linear theory.

The increase in the drag coefficient \( \frac{C_D}{x_0^2} \) of the body between transonic and supersonic flow is greatest for bodies with the point of maximum thickness \( x_0 = 0.35 \), and decreases as the point of maximum thickness moves aft. On the other hand, for the most rearward position of the point of maximum thickness, the drag at transonic speed is already more than 90% of that at supersonic speed.

The flow in the subsonic region of the tail has not been calculated for bodies VI and VII, since the drag can be obtained with sufficient accuracy from experience with body V. The partial drags of the afterbodies at \( M_\infty = 1 \) are therefore given in fig. 10 up to points \( \frac{x}{x_0} \) of the chord of the bodies. They are given for the whole of body V. For bodies VI and VII they are known up to the point where sonic velocity is reached and are extrapolated in the final small region. If the body were broken off far in front of its end, then body VII would give the most favourable drag coefficient (fig. 10). The too-blunt end of the body not only causes the gain to be lost, at about \( \frac{x}{x_0} = 1.435 \), but the total drag becomes considerably worse. For body V, the recovery in drag due to subsonic flow at the tail after the sonic point \( \frac{x}{x_0} = 1.46 \), where the drag is maximum, is clearly apparent, and has been similarly extrapolated for bodies VI and VII. Here, the drag coefficients have been obtained from the velocity on the body, first part of eq. (21). The values of \( C_D \) for bodies VI and VII from this extrapolation are given in table 1.
The transonic similarity rule (see ref. /3/, eq. (47)) applies to equivalent wings

\[ Q_x(L) = 0 : D_{\text{wing}} = D_{\text{eq. body of rev.}} \]  

The conversion for various \( Q_{\text{max}} \) follows for the body of revolution whose drag is equal to that of the wing.

The subcritical Mach numbers are obtained from the linear subsonic flow (cf. chapter 1). They are shown in fig. 11 for the \( T \) bodies by plotting (or \( \delta \), shown dashed) against \( \mu \). For given values of \( \mu \) or \( \delta \), eq. (17), \( \mu^* \) is obtained from eq. (22).

The supercritical Mach number, that of shock wave separation, is given in transonic similarity according to K. Oswatitsch - L. Sjödin /15/ by

\[ M_{\infty}^* > 1 : \mu = \frac{137}{2\pi} \frac{Q_{xx}(0)}{Q_{\text{max}} x^2 \delta} \]  

and can readily be used for equivalent wings.

The subcritical Mach number is greatly dependent upon the manner in which the velocity distribution and \( Q_{xx}(x) \) vary along the chord. Using \( \mu \) it is determined by:

\[ M_{\infty} < 1 : \frac{u(x, \overline{h})}{\pi \tau^2} = -\mu = \frac{\Omega_{ix}(x)}{2\pi \tau^2} \ln \left( \frac{\overline{h}(x)}{\delta} \right) + \frac{1}{2\pi} \frac{Q_{xx}(x)}{\pi \tau^2} \ln \left( \sqrt{1 - M_{\infty}^2} \right) \]  

with \( \Omega_{ix} \) for \( M_{\infty} = 0 \).

For equivalent wings, the critical Mach number is also dependent on the overall shape of the wing along the whole chord, since according to Appendix II the cross-sectional flow (12) of the body of revolution is to be replaced by that of the wing in eq. (26). But this depends on the shape of the cross-section as well as on the local span. The calculation has to be carried out for the given wing.

The perturbation velocity \( \frac{u}{\pi \tau^2} \) is given in fig. 12 to 18 for \( \mu = -1,745 \), for \( \mu = 0 \) (sonic) and \( \mu = 1,745 \) for all bodies of revolution I to VII and their equivalent wings in transonic similarity, plotted along the chord of the body. \( \mu < 0 \) always means subsonic and \( \mu > 0 \) always means supersonic. In addition, a perturbation velocity \( u_{\text{red}} \) will be introduced according to the transonic similarity rule, see eq. (27). For bodies of revolution in supersonic flow it follows with /3/, eq. (29) and the spatial influence at \( M_{\infty} = \sqrt{2} \) that:

\[ \Omega_{\sqrt{2}}(x) \]

\[ u_{\text{red}} = \frac{u(x, \overline{h})}{\pi \tau^2} = \frac{1}{2\pi} \frac{Q_{xx}(x)}{\pi \tau^2} \ln \left( \pi^2 \sqrt{2} \right) + \frac{1}{2\pi} \frac{Q_{xx}(x)}{\pi \tau^2} \ln \left( \frac{\overline{h}(x)}{\delta} \right) \]

In subsonic flow (\( \mu < 0 \)), the right-hand side of the spatial influence at \( M_{\infty} = 0 \); \( \Omega_{ix}(x) \), remains in the first term, and \( \mu \) has to be replaced by \( -\mu \) in the last term. For equivalent wings it follows
quite analogously (from ref. 3, eq. (25) and from eq. (27) here) that:

\[
\frac{u_{\text{red}}}{x_0^2} = \frac{Q_{\text{max}}}{x_0^2} = u(x,0,2) - \Phi_x(x,0,2) + \frac{Q_x(x)}{4\pi} \left[ \ln \frac{Q(x)}{Q_{\text{max}}} - \ln \frac{(Y + 1)Q_{\text{max}}}{x_0^2} \right]
\]

(28)

In this form of the perturbation velocity \( u_{\text{red}} \), only those quantities are stated which are known for equivalent wings. Thus \( \pi x_0^2 \) is replaced by \( \frac{Q_{\text{max}}}{x_0^2} \). Putting the cross-sectional flow of the body of revolution (12) in place of \( \Phi_x(x,0,2) \) of the wing, eq. (28) becomes eq. (27).

Observing the changing scale of \( u_{\text{red}} \) in fig. 12 to 18, it is at once apparent that the perturbation flow on the forebody is practically the same for all bodies in subsonic flow also. As may be conjectured from the result that the transonic drag is half the supersonic drag, the transonic (\( \mu = 0 \)) perturbation flow is approximately half the supersonic (\( \mu > 0 \)) value. Only in the neighbourhood of the nose of the body does the character of the flow at \( \mu = 0 \) approach more closely the subsonic type. For the flow on the afterbody, the maximum of the perturbation velocity for bodies I to V lies farther forward at \( \mu = 0 \) than at \( \mu = +1.745 \). In the case of bodies VI and VII the rise is too great for any statement to be possible. For all afterbodies, the perturbation velocity directly behind the point of maximum thickness is greatest in transonic flow. It attains its smallest value in subsonic flow for bodies I to IV and in supersonic flow for bodies V to VII. The reason for this is that, for the first 4 bodies, the curves for \( \mu = -1.745 \) and \( \mu = +1.745 \) intersect in the immediate neighbourhood of the point of maximum thickness, whereas for bodies V to VII they do not intersect until considerably later.

The transonic perturbation velocities at the rear of the afterbody then lie under those for \( \mu = +1.745 \).

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Kegelige Überschallströmung in Schallnähe.


Table 1:
List of the bodies of revolution

<table>
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<tr>
<th>Body</th>
<th>$\frac{Q(x_0)}{\pi l^2}$</th>
<th>$b_0 \frac{L-x_0}{L}$</th>
<th>$b_2 \frac{L-x_0}{L}$</th>
<th>$b_3 \frac{L-x_0}{L}$</th>
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<th>$M_\infty = \sqrt{2}$</th>
<th>$M_\infty = 1$</th>
<th>$M_\infty = \sqrt{2}$</th>
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<td>3.37</td>
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$CD/\pi l^2$ for forebody, whole bodies, and afterbody.

$x$) numerical integration with the spatial influence

$xx$) " " " " velocity distribution

equation from fig. 10
Table 2.

a) Data for the forebody

\((\tau = 0.146)\)

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<th>(\tilde{E}(x) )</th>
<th>(\frac{\tilde{E}(x) \cdot \Psi(x)}{\pi^2 x_0^2} )</th>
<th>(\frac{\tilde{E}(x) \cdot \Psi(x)}{\pi^2 x_0^2} )</th>
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### b) Datas for the afterbodies

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<th>( \frac{Q(x)}{2\pi\gamma x_0^2} )</th>
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Appendix 1

Chosen Geometrical systematic of the form of bodies of revolution:

1) Forebody: \( D = x \times x \times x \)  
\[ \Omega(x) = 0 \]
\[ \Omega_F(x) = \frac{Q_1(x)}{\pi \times x^2} \]
\[ \Omega_F(x) = \frac{Q_2(x)}{\pi \times x^2} \]
\[ \Omega_F(x) = \frac{Q_3(x)}{\pi \times x^2} \]
\[ x_0 = \frac{Q_4(x)}{\pi \times x^2} \]

2) Afterbody: \( x_0 = x \times x \), \( x = L(x-1) + x_0(2-x) \)
\[ B = L(x) \times x \]

Terms of the forebody:
\[ \delta_{A_0}(x) = (2-x)(1-x)(5-3x) \]
\[ \delta_{A_2}(x) = \frac{1}{2}(1-x)^2 \]
\[ \delta_{A_1}(x) = \frac{3}{2}(1-x)^2 \]
\[ \delta_{A_2}(x) = \frac{3}{2}(1-x)^2 \]
\[ \delta_{A_3}(x) = \frac{3}{2}(1-x)^2 \]
\[ \delta_{A_4}(x) = \frac{3}{2}(1-x)^2 \]

Terms of the afterbody:
\[ \delta_{A_0}(x) = (2-x)(1-x)(5-3x) \]
\[ \delta_{A_2}(x) = \frac{1}{2}(1-x)^2 \]
\[ \delta_{A_1}(x) = \frac{3}{2}(1-x)^2 \]
\[ \delta_{A_2}(x) = \frac{3}{2}(1-x)^2 \]
\[ \delta_{A_3}(x) = \frac{3}{2}(1-x)^2 \]
\[ \delta_{A_4}(x) = \frac{3}{2}(1-x)^2 \]
Cross-sectional area terms and its differential quotients, for the afterbody and the forebody

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Note: The table above shows the cross-sectional area terms and its differential quotients for the afterbody and the forebody. The values are calculated for different values of $X$. The table also includes the 2nd differential quotients of the cross-sectional area for the afterbody and the forebody.
Appendix II

LINEAR THEORY: THE VELOCITY-DISTRIBUTION ON BODIES OF REVOLUTION IN SUBSONIC AND SUPERSONIC FLOW

\[
\frac{u}{\pi \tau^2} = \frac{\varphi_x(x, y, \bar{h}(x))}{\pi \tau^2} + \frac{\Omega_X(x)}{\pi \tau^2} + \frac{1}{2\pi} \frac{Q_{XX}(x)}{\pi \tau^2} \ln \sqrt{M_{\infty}^2 - 1}
\]

with the cross-sectional flow

a) for bodies of revolution:

\[
\frac{\varphi_x(x, \bar{y}, \bar{h}(x))}{\pi \tau^2} = \frac{1}{2\pi} \frac{Q_{XX}(x)}{\pi \tau^2} \ln \frac{\bar{h}(x)}{x_0}
\]

b) for pointed wings:

\[
\frac{\varphi_x(x, \bar{y}, 0)}{\pi \tau^2} = \frac{1}{2\pi} \int \frac{q \left( \frac{x}{x_0}, 1 \right) \ln \left( \frac{y}{x_0}^2 + \left( 1 - \frac{z}{x_0} \right)^2 \right)}{x_0} \, dt
\]

For the spatial influence \( \frac{\Omega_X(x)}{\pi \tau^2} \) and the second derivative \( \frac{Q_{XX}(x)}{\pi \tau^2} \) of the cross-sectional-area \( \frac{Q(x)}{\pi \tau^2} \), see the geometry of the bodies of revolution.

The spatial influence is divided into two parts: the influence of the forebody \( \Omega_{XF}(x) \) and that of the afterbody \( \Omega_{XA}(x) \)

\[
\frac{\Omega_X(x)}{\pi \tau^2} = \frac{\Omega_{XF}(x)}{\pi \tau^2} + \frac{\Omega_{XA}(x)}{\pi \tau^2}
\]

We also distinguish between two intervals of the coordinate \( x \) at the location of the forebody:

\[0 \leq x \leq x_0; \quad \xi = \frac{x}{x_0}; \quad x = \xi x_0.\]

and at the location of the afterbody:

\[x_0 \leq x \leq L; \quad X = 1 + \frac{x - x_0}{L - x_0}; \quad x = L(X - 1) + x_0(2 - X).\]
Spatial influence of the subsonic flow ($M_a = 0$)

\[
0 = x = x_0 : \quad (0 \leq x \leq 1)
\]

\[
\frac{\partial \psi(x)}{\partial x} = -\frac{3}{2\pi} \left[ 1 - 6(1 - 2x) \ln(2x - 1) - \frac{1}{3} - (1 - 2x)(2 - 9x) \right] +
\]

\[
\int_{x_0}^{x} \frac{\partial \psi(x)}{\partial x} \, dx = \frac{2}{3\pi} \left[ 1 - 6(1 - 2x) \ln(2x - 1) - \frac{1}{3} - (1 - 2x)(2 - 9x) \right] +
\]

\[
= \frac{3}{2\pi} \left[ 1 - 6(1 - 2x) \ln(2x - 1) - \frac{1}{3} - (1 - 2x)(2 - 9x) \right] +
\]

\[
\frac{\partial \psi(x)}{\partial x} = -\frac{3}{2\pi} \left[ 1 - 6(1 - 2x) \ln(2x - 1) - \frac{1}{3} - (1 - 2x)(2 - 9x) \right] +
\]

\[
\int_{x_0}^{x} \frac{\partial \psi(x)}{\partial x} \, dx = \frac{2}{3\pi} \left[ 1 - 6(1 - 2x) \ln(2x - 1) - \frac{1}{3} - (1 - 2x)(2 - 9x) \right] +
\]

\[
= \frac{3}{2\pi} \left[ 1 - 6(1 - 2x) \ln(2x - 1) - \frac{1}{3} - (1 - 2x)(2 - 9x) \right] +
\]

Spatial influence of the supersonic flow ($M_a = V_F$)

\[
0 = x = x_0 : \quad (0 \leq x \leq 1)
\]

\[
\frac{\partial \psi(x)}{\partial x} = -\frac{3}{2\pi} \left[ 2(1 - 2x)(1 - 3x) \ln(2x + 8x - 9x^2) \right] -
\]

\[
\int_{x_0}^{x} \frac{\partial \psi(x)}{\partial x} \, dx = \frac{2}{3\pi} \left[ 2(1 - 2x)(1 - 3x) \ln(2x + 8x - 9x^2) \right] -
\]

\[
= \frac{3}{2\pi} \left[ 2(1 - 2x)(1 - 3x) \ln(2x + 8x - 9x^2) \right] -
\]

\[
\frac{\partial \psi(x)}{\partial x} = -\frac{3}{2\pi} \left[ 2(1 - 2x)(1 - 3x) \ln(2x + 8x - 9x^2) \right] -
\]

\[
\int_{x_0}^{x} \frac{\partial \psi(x)}{\partial x} \, dx = \frac{2}{3\pi} \left[ 2(1 - 2x)(1 - 3x) \ln(2x + 8x - 9x^2) \right] -
\]

\[
= \frac{3}{2\pi} \left[ 2(1 - 2x)(1 - 3x) \ln(2x + 8x - 9x^2) \right] -
\]

\[
\frac{\partial \psi(x)}{\partial x} = -\frac{3}{2\pi} \left[ 2(1 - 2x)(1 - 3x) \ln(2x + 8x - 9x^2) \right] -
\]

\[
\int_{x_0}^{x} \frac{\partial \psi(x)}{\partial x} \, dx = \frac{2}{3\pi} \left[ 2(1 - 2x)(1 - 3x) \ln(2x + 8x - 9x^2) \right] -
\]

\[
= \frac{3}{2\pi} \left[ 2(1 - 2x)(1 - 3x) \ln(2x + 8x - 9x^2) \right] -
\]
Diagramm A, to Appendix II:
Flow influences after the forebody
Diagram B, in Appendix II: Subsonic flow influences before the afterbody.
Appendix III

CONTRIBUTION OF THE PRESSURE TO THE DRAG (linear theory, $M_\infty = V_2$)

$$\frac{(D)\sqrt{2}}{\frac{9 \rho \mu^2}{2 \infty} \max} = (C_D)_{V2} = (C_D)_{V2}^I + (C_D)_{V2}^II + (C_D)_{V2}^III$$

Drag coefficient of the forebody:

$$\frac{1}{t^2}(C_D)^I_{V2} = 6 + \frac{1}{2} \frac{Q_{xx}(x_0)}{\pi \tau^2} + \frac{1}{8} \frac{Q_{xx}(x_0)}{\pi \tau^2}^2$$

Influence of the forebody on the afterbody to the drag:

$$\frac{1}{t^2}(C_D)^II_{V2} = k_{00} + k_{10} \frac{Q_{xx}(x_0)}{\pi \tau^2} + k_{11} \frac{Q_{xx}(x_0)}{\pi \tau^2}^2 + [k_{20} + k_{21} b_2 + k_{22} b_3] b_2 + [k_{30} + k_{31} b_2] b_3$$

The constants of $(C_D)^II_{V2}$

<table>
<thead>
<tr>
<th>$\frac{x_0}{L}$</th>
<th>$k_0$</th>
<th>$k_{10}$</th>
<th>$k_{11}$</th>
<th>$k_{20}$</th>
<th>$k_{21}$</th>
<th>$k_{31}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.2334</td>
<td>-0.20479</td>
<td>+0.01121</td>
<td>-0.0142</td>
<td>-0.00845</td>
<td>+0.00627</td>
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<tr>
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<td>0.4797</td>
<td>-0.13499</td>
<td>+0.00882</td>
<td>-0.0234</td>
<td>-0.0129</td>
<td>+0.001385</td>
</tr>
<tr>
<td>0.50</td>
<td>0.8795</td>
<td>-0.08956</td>
<td>+0.00543</td>
<td>-0.0347</td>
<td>-0.0183</td>
<td>+0.00325</td>
</tr>
<tr>
<td>0.65</td>
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<td>-0.0217</td>
<td>+0.0064</td>
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<tr>
<td>0.75</td>
<td>2.1378</td>
<td>-0.21351</td>
<td>+0.00153</td>
<td>-0.0445</td>
<td>-0.0223</td>
<td>+0.00924</td>
</tr>
</tbody>
</table>

* Interpolated values

Influence of the afterbody to the drag:

$$\frac{1}{t^2}(C_D)^III_{V2} = \left(\frac{x_6}{L-x_0}\right)^2 \left[6 + \frac{1}{2} b_1 (1 + \frac{1}{12} b_1) + \frac{1}{3} b_2 (1 + \frac{1}{4} b_1 + \frac{3}{16} b_2) + \frac{1}{4} b_3 (1 + \frac{1}{6} b_1 + \frac{1}{6} b_2 + \frac{1}{20} b_3)\right]$$

$$b_1 = \left(\frac{L-x_0}{x_6}\right)^2 \frac{Q_{xx}(x_6)}{\pi \tau^2}$$

$b_2$ and $b_3$ are parameters of the geometry of the body.
Appendix IV

APPROXIMATION OF THE SUBSONIC STAGNATION FLOW NEAR THE TRAILING EDGE.

The characteristics constructions for all the bodies calculated gave a delay in the occurrence of supersonic flow (fig. 6) and the Mach lines intersected before sonic velocity was obtained. While the velocity on the actual body was continuous, shocks were formed at a short distance from the surface, with nearly sonic velocity behind the shock front.

Fundamentally, the subsonic flow at the tail could cause such a strong stagnation that the supersonic flow would once more be disturbed upstream and overlapped by an almost normal shock. The latter is however improbable. For at a distance from the axis of the body the tail shock certainly approaches the free stream velocity, i.e. sonic velocity. Likewise, the stagnation at the trailing edge certainly diminishes rapidly outwards, like that at the extreme nose. The trailing edge stagnation arises essentially through deflection of the convergent flow to approximately parallel flow. It is thus to be assumed that maximum deflection takes place in the shock. This would also be well satisfied by a shock leading to sonic velocity.

The assumption of a trailing-edge shock leading to sonic velocity is thus largely correct for what actually occurs and is certainly a sufficiently accurate approximation for calculation of the drag.

In other respects, the trailing-edge flow in the linear supersonic flow is not accurately determined.

With the exception of the flow at the trailing-edges of bodies VI and VII, which could be neglected because of their small extent, a trailing-edge shock leading everywhere to sonic flow was assumed for all bodies. The first intersection of the Mach lines near the body leading approximately to sonic velocity then serves as the beginning of the shock. This gives a sonic line on which vy is also known from the shock condition. Now, since at sonic velocity vy does not vary with y, the derivative of vy with respect to x on the sonic curve is equal to the derivative at constant y: \( \frac{\partial vy}{\partial x} \).

Also, \( \frac{\partial vy}{\partial x} \sim k(x) \) is known on the body. Thus \( \frac{\partial vy}{\partial x} \) on perpendicular straight lines \( (x = \text{const.}) \) can be approximated very well by linear interpolation between the values on the body and the trailing-edge shock.

However, \( u \) can also be easily obtained using the equation for irrotational flow:

\[
\frac{\partial u}{\partial y} = \frac{1}{y} \frac{\partial vy}{\partial x}
\]

and integrating over y. Since the sonic line is fixed, \( u \) on the body and the spatial influence on the axis are also given.
Distribution of the body-radius and of the first differential quotient of the cross-sectional area over the body length.

Fig. 1

Distribution of the 2nd differential quotient of the cross-sectional area along the axis of the body.

Fig. 2
Fig. 3. Contribution of the pressure to the drag in supersonic flow.

- \( C_{D_{1/2}} \), \( C_{D_{6/2}} \)
- \( \rho \) values for the bodies at \( \mu = 1.745 \)
- \( C_{D_{1/2}} \) for the bodies at \( \mu = 0 \)
- Geometrical limit

\( b_2 \) and \( b_3 \) are fixed by the same \( \Omega_{xx}(x_0) \) for forebody and afterbody

- No condition for \( b_2 \) and \( b_3 \)
Equivalent wing to body I

\[ \begin{align*}
\text{a) wing plane} \\
X = a \times X_0 \\
0 < X < 15 \\
0 < Z < 0
\end{align*} \]

Equivalent wing to body III

\[ \begin{align*}
\text{b) cross-sections} \\
\frac{h(x,z)}{h(x_0,0)} \\
0 < X < 15 \\
0 < Z < 0
\end{align*} \]

Equivalent delta-wing to the bodies of revolution I and V

\[ \begin{align*}
\text{c) maximum thickness-distribution along the wing-root.} \\
\frac{Q(x)}{Q_{\text{max}}} = x
\end{align*} \]

Equivalent swallow-tail wing to the bodies of revolution I and III.

\[ \begin{align*}
\text{d) maximum thickness-distribution and longitudinal section in the wing-root.} \\
\frac{Q(x)}{Q_{\text{max}}} = x
\end{align*} \]
Fig. 5. Mach lines for $M_\infty = 1$ at body I

$M_\infty = 1.1971 \; ; \; \mu = 2.015$

Body I (spindle), $\tau = \delta = 0.146$

X not linear Characteristik method /14/ 
O linear Characteristik method (Sauer-Heinz)

Fig. 7 Comparison of the results of the linearized and non-linearized method of characteristics.

Fig. 8 Pressure distribution at $M_\infty = 1$ for 3 bodies.
Fig. 9 The drag as a function of the Mach parameter $\mu$.

Critical Mach-Numbers dependent on the maximum of the thickness of the body.

Fig. 10 Drag contributions of the three short afterbodies.

Fig. 11 Critical Mach-Numbers dependent on the maximum of the thickness of the body.
The distribution of the reduced $u$-component on the seven bodies at 3 different values of the Mach parameter $\mu$.