A Method for Designing Lifting Configurations For High Supersonic Speeds, Using Axisymmetric Flow Fields*

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Summary: A method is proposed which may be used as a basis for the rational design of lifting vehicles intended to cruise at speeds between \( M = 4 \) and 7. In order to construct shapes whose behaviour in an inviscid fluid stream may be calculated exactly, stream surfaces chosen according to certain rules from axisymmetric flow fields are replaced by solid boundaries. Combinations of such stream surfaces may be built-up piecemeal into complete aircraft configurations, and it is shown that the method is sufficiently flexible to allow considerable freedom of choice in the disposition of lift and volume. Methods are indicated whereby the lifting efficiencies of such shapes may be investigated and, to some extent, optimised.


1. Introduction. In this paper a method, based on the use of axisymmetric flow fields, for designing a possible type of cruise vehicle for the approximate Mach number range \( M = 4 \) to 7 is described.

It is desired to obtain a configuration shape with integrated lifting surface, stowage volume and propulsion unit as in Fig. 1, and the object has been to obtain a vehicle which will have satisfactory (fixed geometry) low speed, low supersonic speed, and cruise Mach number performance. The general design philosophy of such “integrated shapes” has been discussed by Seddon [1] and Kitcherman [2]. Since the slender wing at \( M = 2 \) is already near the limit of sweep for low speed performance, the vehicle envisaged will not be aerodynamically slender at the cruise Mach number. Accepting this, it has been designed so that the lower surface shock wave is effectively attached at the leading edge in the cruise condition. At a Mach number \( M = 4 \), which has been used throughout the paper in the illustrative examples, the chosen planform has leading edge sweep approximately the same as that of an \( M = 2 \) slender wing. At transonic and low supersonic speeds the vehicle will behave as a slender wing (although the streamwise area distribution will be quite different from that of an \( M = 2 \) to 3 slender wing supersonic transport) and as the cruise Mach number is approached the lower surface shock wave moves up to, and finally attaches at, the swept leading edge.

The design philosophy followed is basically that of finding a shape with a theoretically known, and acceptable, flow field in the cruise condition. In this way required lift coefficient and centre of pressure, for example, can be obtained, and the shock wave position and pressure gradients controlled in such a way that the theoretical (non-viscous) model is likely to be a good approximation to the actual flow field achieved in practice. A method is required containing a sufficient number of free parameters to allow freedom of choice so that performance, lift to drag ratio for example, although not theoretically an optimum, is reasonably good. At low supersonic speeds linearised theory is a useful tool in this respect; a suitable wing loading is chosen and then a wing camber shape can be designed to support this loading. In the high supersonic Mach number range linear theory is no longer sufficiently accurate and has in the present method been replaced by the process of making use of parts of known axisymmetric flow fields, replacing stream surfaces in these relatively simple flows by suitably chosen solid surfaces. The restriction to axisymmetric flow fields has been made because these can be calculated relatively easily using the method of characteristics. Momentum methods are used to determine which of these shapes have good lift to drag ratios.

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Since the shock wave is attached at the leading edge in the cruise condition the upper and lower surfaces of the configuration have independent flow fields and can thus be designed separately. At high supersonic speeds the wing lower surface becomes increasingly important for the production of lift, as illustrated in Fig. 2. The suction on the upper surface are limited as a complete vacuum condition is approached. In the present method, then, the lower surface is regarded as the primary lifting element and is designed to have known lift coefficient and centre of pressure. The major condition in the design of the upper surface is that shock waves are confined to the leading edge and trailing edge. If not specially controlled, shock waves tend to appear above the wing surface in the upper surface flow field [3], as illustrated in Fig. 3. The expansion just downstream of the leading edge tends to be followed by a shock wave which turns the flow in the direction of the free stream again. In the present design method such shock waves are removed, essentially by spreading the expansion over the whole wing in a controlled manner.

![Integrated shape providing lift and volume](image)

**Fig. 1.** Integrated Vehicle for High Supersonic Speeds (schematic).

![Propotional Contributions of Upper and Lower Surfaces to the Lift of a Two-Dimensional Flat Plate](image)

**Fig. 2.** Proportional Contributions of Upper and Lower Surfaces to the Lift of a Two-Dimensional Flat Plate.

The propulsion unit is a major feature of a high supersonic speed cruise vehicle, and in this paper it has been assumed that a mixed turboramjet system with subsonic ramjet combustion in the cruise condition is used. The intake and exit areas, which from the point of view of external geometry are the main propulsion unit parameters, are determined by the associated mass flows in the cruise condition. These parameters are fundamental to the design of the complete configuration. For example the relation between the upper and lower surfaces towards the rear is largely determined by the distribution of base area assumed to be filled by the ramjet exit (Fig. 1).

The basic arrangement of the paper is as follows. Section 2 is concerned with the design of lower surfaces. Two basic shapes are illustrated, derived from the flow field past a non-lifting cone. This approach is then extended to include the flow field past a cone-cylinder combination, and it is suggested that the flow past a general axisymmetric ogive will be used in future work. Section 3 contains illustrations of the application of the method to the design of upper surfaces. In Section 4 it is shown how momentum theory can be used as a basic tool in the understanding of the relation between wing shapes and lift to drag ratios. Finally, in Section 5, it is shown how upper and lower surfaces can be combined into a complete configuration, including propulsion unit.

In practice, it will be necessary to make slight modifications to shapes derived using the present method and for which the exact inviscid flow field is known. For example, leading edges will have to be rounded to take account of heating. The philosophy adopted in this respect is that small perturbations of the basic shapes, adopted for such practical reasons, will only introduce small changes in the predicted wing properties. The actual effect of such perturbations will be determined by experiment.

This paper is intended only as a summary of the method described: the technical details are elaborated more fully in Refs. 4 to 7.

### 2. Design of Lower Surfaces.

#### a) Summary of previous work.

At high supersonic speeds the flow field about a lifting wing contains a strong shock wave on the high pressure side. Owing to the strongly non-linear nature of the flow field the most feasible design method giving a configuration with theoretically predictable properties appears to lie in the use of basic simple known flow fields, making use of the fact that stream surfaces can be replaced by solid surfaces. An example of this procedure has been described by Nonuseller [8, 9] who takes the flow past a two-dimensional
wedge as basic flow field. The leading edge of the configuration is prescribed as a curve on the plane oblique shock wave and the lower surface is obtained by replacing the stream surface through the leading edge by a solid surface. Examples are presented in Fig. 4. In Fig. 4(a) the leading edge 'curve' is prescribed as an 'inverted — V'. The upper surface has been drawn parallel to the free stream, but in fact is arbitrary provided it does not cause the lower surface shock wave to become detached from the leading edge. In Fig. 4(b) it is shown how two simple flow fields can be combined to give a more general configuration. Properties of such configurations are described in Refs. 10 and 11. In each case the pressure coefficient is constant on the lower surface.

In the following sections it is shown how this basic idea can be extended to give a more general class of useful lower surface shapes by taking a known axisymmetric flow field as the starting point.

b) Use of flow field of non-lifting cone. a) Introductory remarks. In the present section it is shown how the flow past a non-lifting circular cone can be used as the basic flow field [4], and the resulting wing lower surface shapes are discussed. The leading edge curve now lies on the conical shock wave (Fig. 5) and the lower surface is obtained, as before, by replacing the stream surface through the leading edge by a solid surface. The pressure coefficient on the surface is no longer constant but the variation is not large.

Two basic types of configuration have been considered, depending upon whether or not the cone apex lies on the prescribed leading edge. The case where the cone apex becomes the apex of the configuration (to be called a 'Type A configuration') is discussed in Section β below. The simplest example of this type is the half-cone combined with flat plate wing at zero incidence. The more general shapes considered combine part of a cone with a wing which may be at incidence.

In the other type of configuration (to be called Type B) the apex lies off the basic cone axis (Section γ below). The principal difference between the Type A and Type B configurations is that in the former case wings and body are distinguishable as separate entities on the lower surface (the body being part of the basic cone) whereas in the latter they have been 'integrated'. Of course, by taking the apex of the configuration near to the cone apex a smooth transition between the two types can be obtained.

In evaluating the pressure forces on a surface it has been found convenient to take free stream static $P_\infty$ as the zero level for pressure. Thus for the lower surface we define the lift force

$$L = \int (p - P_\infty) \, dS,$$
where the integral is taken over the projected planform area $S_1$ and the pressure drag

$$D_p = \int_S (p - p_\infty) dS,$$

where the integral is taken over the front projection $S_2$. The sum of the drags $D_p$ of the upper and lower surfaces then becomes the pressure drag of the complete configuration with a base pressure assumed equal to free stream static. Moreover, the lift coefficient of a surface is now taken to be the average pressure coefficient over the projected planform area. Provided that the same convention is used on both upper and lower surfaces the terms involving $p_\infty$ in the lift $L$ cancel.

Since the variation of pressure coefficient on the lower surface is not large, choice of lower surface lift coefficient (which is an average pressure coefficient) corresponds approximately to choice of basic cone angle. This is a useful starting point in the present design method.

b) Type A configurations. In this section configurations are discussed whose apex lies at the apex of the basic cone.

1) Design method. Fig. 6 illustrates the basic flow field past a cone of semi-angle $\alpha$. The shock wave angle is $\sigma$, and the flow deflection at the shock wave $\delta$. Fig. 6 illustrates the construction of a typical configuration in front view. Firstly, the lines $OA$, $OB$ are drawn on the shock wave front projection. The choice of the angle $AOB$ will be discussed later; here it has been arbitrarily chosen to be $90^\circ$. The curve $OC$, concave upwards as illustrated, is next drawn to be tangent to $OA$ at $O$. This curve, on the shock wave, is now defined to be the leading edge of the configuration, and the lower surface is to be found by tracing the stream surface through $OC$. A point $P$ is chosen on the leading edge $OC$, and, in Fig. 6(b), the radial line $OP$ is drawn, representing the radial plane through the cone axis and $P$. Owing to the axial symmetry of the basic flow field the streamline originating at $P$ ends at the point $Q$. The distance of $Q$ from the cone axis becomes the distance $OQ$ in Fig. 6(b), thus determining the point $Q$. By repeating this construction for a number of points $P$ on the leading edge the trailing edge $CD$ is found. In the same way the shape of any section can be found graphically and the shape of the complete undersurface determined.

Viewed from the front (Fig. 6(b)) the configuration is seen to consist of part of the cone (shaded) and a wing, the area $OCD$ corresponding to the underside of the wing. The purpose of making the curve $OC$ concave upwards was in fact to obtain a wing at a positive angle of attack.

The plan and side views of the lower surface are constructed by projecting the front view. Pressure coefficients at all points on the lower surface are obtained directly from the basic flow field data, and numerical integrations of pressure coefficient over the plan and front projections respectively give lift and drag for the lower surface and hence $L/D_p$ and $C_L$.

ii) Relations between leading edge and trailing edge shapes. Some analytical relationships are now presented which allow a more exact determination of the shape of the wing lower surface relative to the leading edge. Results are given for the wing trailing edge, but apply equally...
to all lower surface sections. Proofs are omitted here but are given in a previous paper [4]. An entirely analytic approach has been presented by Woods [12].

The first relationship is between the shape of the leading edge at the apex and the shape of the trailing edge at the wing root (see Fig. 6(b)).

1. If the trailing edge is not tangential to the cone at the wing root the radius of curvature of the front projection of the leading edge at the apex is infinite.

2. If the radius of curvature of the front projection of the leading edge at the apex is finite (see Fig. 7), then the trailing edge is tangential to the cone at the wing root and the radii of curvature $R_1$ of the leading edge projection at the apex, and $R_2$, of the trailing edge projection at the wing root, are related by the equation

$$
\frac{4 R_1 R_2}{r^2 (r + R_2)} = \left( \frac{\rho}{\rho_{\infty}} \right) \left( \frac{v}{v_{\infty}} \right) \cos \alpha,
$$

where $r$ is the radius of the basic cone at the trailing edge, $\alpha$ is the cone semi-angle, $\rho_{\infty}$ and $v_{\infty}$ are respectively density and velocity in the free stream and $\rho$, $v$ are density and velocity at the surface of the cone.

The second relationship is between the shapes of the leading and trailing edges at the wing tip. Suppose that at the wing tip $C$ the front projections of the leading and trailing edges have tangents making angles $\gamma_1$ and $\gamma_2$, respectively with the tangent to the shock wave (Fig. 8), then:

$$
\tan \gamma_1 = \tan \sigma - \tan \delta,
$$

where $\sigma$ is the shock semi-angle, and $\delta$ is the flow deflection at the shock.

iii) Example of Type A configuration. In this example (Fig. 9) the basic flow field is taken to be that past a 10° semi-angle cone at $M = 4$. The angle $AOB$ (Fig. 6(b)) has been taken to be 90°, and the front projection of the leading edge (OPC) chosen as an arc of a circle. The upper surface has been constructed by taking generators parallel to the free stream through the leading edge; the lower surface $C_L$ and $L/D_p$ values thus become overall values for the configuration. Since the front projection of the leading edge has non-zero curvature at $O$, it follows from Section ii) that the trailing edge is tangential to the cone at the wing root. Equation (1) has been used in constructing the shape of the lower surface at the wing root, and equation (2) in constructing the tip angle at each section.

Section AA (Fig. 9) illustrates the limiting form that all Type A configurations take near the apex. In particular it can be seen that the wing thickness tends to zero. In practice thickness can be obtained by adding a small amount of volume on the upper surface, the only necessary condition being that the resulting compression on the upper surface is not large enough to cause shock detachment. When the upper surface is parallel to the free stream the lower surface force data give overall values: $C_L = 0.075$, $L/D_p = 8.1$. 

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**Fig. 8. Relation Between Shapes of Leading and Trailing Edges at Wing Tip.**

**Fig. 7. Relation Between Shapes of Leading and Trailing Edges at Wing Root.**
A variable configuration, of which the above is a special case, can be obtained by varying the angle $AOB$ (Fig. 10) keeping the shape of the leading edge $OC$, relative to $OA$, constant. The resulting variations in $L/D_p$ and $C_L$ are discussed in the following section.

**Fig. 9. Type A Example.**

iv) Typical variation in the lift to pressure-drag ratio. In order to provide a guide for the choice of leading edge position the variations in $L/D_p$ for a set of configurations with fixed leading edge shape have been evaluated. The example already described (Fig. 9) is, in fact, a member of the set, and the various configuration shapes are generated by keeping the shape of the leading edge $OC$ (Fig. 10) fixed relative to the ray $OA$ and varying the angle $AOB$.

Denoting the angle $AOB$ by $2\psi$ (Fig. 10) we can evaluate the forces on a configuration as follows. The forces on the wing parallel and perpendicular to the ray $OA$, $F_1$ and $F_2$ respectively, can be found by integrating the pressure coefficient over corresponding projections of the wing. The lift, for any value of $\psi$ is then found by summing the components of $F_1$ and $F_2$ in the vertical direction. The drag of the wing is independent of the orientation of the ray $OA$. Since the pressure coefficient on the cone is constant, the lift and drag on the shaded portion of cone (Fig. 10) are simply proportional to $\sin \psi$ and $\psi$ respectively. The lift-drag ratio can thus be found as a function of $\psi$. The values of $L/D_p$ and also the values of $C_L$ which are obtained similarly, are presented in Fig. 11.

As $\psi$ increases from zero the value of $L/D_p$ decreases, but at the same time the distribution of useful volume improves. As pointed out earlier, since the pressure coefficient on the lower surface is approximately constant the value of $C_L$ (based on projected planform area) is largely independent of $\psi$. The results presented are for pressure forces only, but the effects of skin friction could easily be incorporated into this kind of analysis.

**Fig. 10. Evaluation of $L/D_p$ for Variable $\psi$.**

γ) Type B configurations. i) General remarks. An example of this type of wing can be constructed as follows (see Fig. 12). The upper surface comprises two intersecting streamwise planes. The upper surface intersects the conical shock wave associated with a 16° semi-angle cone at $M = 4$, and the leading edge is defined as the curve of intersection. The lower surface of the configuration is then obtained by replacing the stream surface through the leading edge in the cone.
flow field by a solid surface. The resulting shape is a geometrically slender wing (aspect ratio approximately 1.6). Integration of the pressure coefficient over the planform and front projections of the wing gives the lower surface data: $C_L = 0.170$, $L/D_p = 4.8$. Owing to the sharp apex there is a ridge on the centreline of the lower surface (as in 'Nonseelel wings'). The magnitude of the ridge decreases as distance from the apex increases, as shown (Fig. 12). An analytical expression giving the variation in the lower surface 'ridge angle' is described in the following section. The method of Section β ii is equally applicable to Type B configurations at the wing tip.

Near the apex, the Type B configuration (Fig. 12) becomes in the limit a Nonseelel wing (Fig. 4(a)). An advantage of the Type B configuration is the relatively flat undersurface which may be obtained towards the rear. Having a nearly flat undersurface with a flow field known in depth below it is useful in connection with the addition of a propulsion unit. This application is taken up again in Section β below.

ii) Variation in lower surface 'ridge angle'. We now quote a result giving the variation in lower surface 'ridge angle' corresponding to a given leading edge shape at the (pointed) apex. The result is quoted for the value of the ridge angle at the trailing edge, but is equally applicable
to all streamwise stations. The notation used is illustrated in Fig. 13 (in which the size of the ridge angle has been exaggerated), the angle $\xi$ defining the leading edge anhedral at the apex and the angle $\zeta$ defining the "ridge angle" in front projection. Then it can be shown [4] that

$$\tan \zeta = \frac{k(x)}{\tan \xi}$$

where the prime denotes differentiation with respect to $x$ and the function $k(x)$, which defines streamline shape and depends only on free stream Mach number and cone angle, is illustrated in Fig. 14. It can easily be verified that the flow deflection at a point $x$ on the streamline is $k'(x)\tan \sigma$. In fact, a simple generalisation of the above equation can be used to describe the entire distribution of spanwise slope of any cross-section in terms of the leading edge shape.

\[ \tan \zeta = \frac{k(x)}{\tan \xi} \]

Fig. 13. Illustration of Lower Surface "Ridge Angle".

Experimental results. The shape illustrated in Fig. 12 has been tested in the Mach 4 wind tunnel facility at R.A.E. Bedford. A side view of the tunnel model using schlieren photography is shown in Fig. 15. The model is in the design attitude, and Fig. 15 (with other schlieren photographs taken from different directions) confirm the shock wave to be in the theoretically predicted position.

The surface streamlines are illustrated in Fig. 16 by an oil flow technique. The line $aa$ is the cone axis. As the flow is axisymmetric, $aa$ is coplanar with a theoretical streamline on the wing.

The theoretical flow is conical and constant pressure surfaces are cones with $aa$ as axis. Their intersection with the surface gives isobars. Pressure holes on three such isobars, and experimental pressure are shown in Fig. 17. The small deviations from the theoretical values are within the limits of experimental error.

Use of general axisymmetric flow fields. a) Introduction. A disadvantage in the use of a cone flow field as a basis of design for the lower surface is that it may not provide the right relationship between centre of volume (used loosely as an equivalent to "centre of gravity"), centre of pressure and aerodynamic centre. This can be avoided by making use of a more general axisymmetric basic flow field in which the initial compression is followed by an expansion, in this way obtaining lower pressures over the aft part of the wing. Another reason for requiring such
types of pressure distribution is that they are closer to those predicted by linear theory to give optimum lift to drag ratios.

A further disadvantage of a basic cone flow field is that the wing lower surface slope takes its smallest values just behind the shock wave. At low values of wing lift coefficient (resulting from the use of a basic cone of small angle) the angle of incidence just behind the shock wave is only a small fraction of the average lower surface angle of incidence. This property adversely affects the offdesign performance of the wing in that a small decrease in the wing angle of attack causes the local incidence at the leading edge to be reduced to zero or even take on negative values and as a result the shock wave moves inboard. This phenomenon can be avoided by using an axisymmetric flow field, with a stronger compression just behind the shock wave, such as is generated by an ogive shaped body.

In Section $\beta$ below it is described how a cone-cylinder combination has been used to generate the basic flow field thus obtaining a wing with the centre of pressure in a reasonable position. In Section $\gamma$ below the use of the flow field past a general ogive shaped body is discussed.

$\beta$) Cone-cylinder combination. The flow past a cone-cylinder combination is illustrated in Fig. 18. The leading edge of the wing is chosen to lie entirely on the conical part of the shock wave. In this way the lower surface shape and pressure distribution can be found by using only the homentropic part of the flow field, thus simplifying numerical computation using the method of characteristics. In order to obtain maximum amount of expansion over the aft part of the under-

surface the wing tip is chosen on the downstream limit of the conical part of the shock wave. The wing undersurface is taken to be the stream surface through the leading edge. In Fig. 19 a particular example is illustrated. The curve $LT$ (Fig. 19) is the upstream limit of the region affected by the expansion. Also illustrated is the centre of pressure.

The dashed curve in Section EE (Fig. 19) shows the surface obtained from the same cone flow field (and same leading edge) without any subsequent expansion.
The cone-cylinder design has a theoretical performance given by $C_L = 0.076$, $L/D_p = 6.5$.

7) Use of general ogive. As explained in Section x above there are advantages to be gained in using as basic flow field the flow past a general ogive-shaped body. Since a major part of the method consists of tracing stream surfaces in the flow field, it is advantageous, in computing the flow past the ogive, to use a computing mesh in which streamlines play a basic part. Such a computing scheme has been described by Walden and Hottie [13] who use a system of coordinates consisting of streamlines and outgoing characteristics. The governing equations are solved numerically by finite difference techniques. Once the flow field has been computed, generalisations of equations (1) and (2) will be found to assist in determining the wing shapes.

![Fig. 17. Pressure Coefficients along Isobars (with Boundary Layer Correction).](image)

![Fig. 18. Flow Round Cone-Cylinder at Zero Incidence.](image)

d) Integration of propulsive system. It was stated in the Introduction (Section 1) that a typical propulsive system for the type of vehicle under consideration comprises a turbo-ramjet combination in which the ramjet is used in the high supersonic speed cruise condition.

It is now shown how the methods of lower surface design already described can be simply modified to include the external geometry of such a propulsive system.

As a particular example the lower surface obtained from a basic cone-cylinder flow field (Fig. 19) is considered. The intake is chosen to lie in the high pressure region at the 0.53 chord position. The intake area is determined by the ramjet system used and its shape is chosen to be as illustrated in Fig. 20 (Section CC). Since the flow into the intake is assumed to be completely swallowed the flow field external to the propulsion unit can be prescribed completely independently. Having decided upon the intake geometry we may choose the outside of the propulsion unit to be a streamsurface
conforming to the undersurface flow field (Fig. 20). In this way the propulsion unit is added without perturbing the external flow field previously chosen. The pressure distribution on the outside of the propulsion unit is known from the basic flow field, and the modification to the lower surface total lift resulting from the addition of the propulsion unit can easily be found.

The design of the propulsion system exit involves matching with the wing upper surface and will be described in Section 5.

3. Design of Upper Surfaces. a) Introduction. The basic method of starting from a known flow field and replacing suitably chosen streamsurfaces by solid surfaces is applied in the present section to the design of wing upper surfaces.

Two-dimensional expansions have been discussed by Flower [14] and used to design lifting upper surfaces whose leading edges have positive dihedral. In the present study attention has been given to the use of axisymmetric flow fields and upper surfaces have been designed which can be conveniently combined with the lower surfaces previously discussed (Section 2) so as to give complete configurations. Attention is restricted to homentropic flows such as occur on an afterbody attached to a circular cylinder which extends to infinity upstream (Fig. 21). Only that part of the flow field upstream of the tail shock is used. The pressures in the basic flow field will normally be below free-stream static. However, the use of a flow field in which the initial expansion is followed by a homentropic recompression is not ruled out. The shock waves on the resulting upper surface of the wing will in this way be confined to the (supersonic) trailing edge. This is in contrast to the type of flow field which occurs, for example, on the upper surface of a flat delta wing with super-
sonic leading edges at high supersonic speeds. In this case the flow field is as illustrated in Fig. 3. The streamline pattern on the upper surface shows that the initial expansion is associated with an inward flow towards the centre line. The recompression associated with the turning of the streamlines back towards the free-stream direction takes place in part through a shock wave. In the present method of design the upper surface expansion is controlled in such a way that shock waves on the wing upper surface do not occur.

b) Details of the method. In the case of lower surface design, in which the basic flow field contains an initial compression through a shock wave, the wing leading edge is taken to lie on the shock wave. In this way it is ensured that shock waves do not lie actually on the wing lower surface but are confined to the leading edge. In the case of upper surface design, in which the basic flow field begins with an initial expansion, the leading edge can be chosen to lie either on the upstream Mach cone of the axisymmetric expansion or upstream (Fig. 22) of this Mach cone. The wing upper surface is then obtained by replacing the stream surface through the leading edge by a solid surface. In the case where the leading edge lies upstream of the bounding Mach cone the wing upper surface will initially be parallel to the free stream and the upper surface expansion will begin at the boundary TLT (Fig. 22) where this Mach cone is intersected. Note that this boundary is independent of the strength of the expansion. The region TAT' is parallel to the free stream.

Fig. 23 illustrates a particular example. The lower surface of the configuration in this figure is a Type B cone flow surface.

c) Use of two independent basic expansion flows. It has been found useful (see Section 5) to extend the method of upper surface design just described by using two independent axisymmetric expansion flows, each one defining half of the upper wing surface. The technique for achieving this is illustrated in Fig. 24. A Mach cone having its axis XX' is supposed to contain some known expansion flow. Upstream of this cone a broken line LMN is drawn, and the stream surface through LMN traced. As in Section b above, this surface is initially parallel to the free stream, but curves.
towards $XX'$ after intersecting the Mach cone along $FHIT$. Any line in the surface on or ahead of $FHIT$ may be chosen as a leading edge, for example $FIJMT$, and a solid surface such as $FHMJTBF$ will support the design expansion flow. This solid surface may now be joined along the plane $FLMI$ to its mirror image in this plane (Figs. 24(b) and 29) and the resulting shape regarded as forming the complete upper surface of a wing, plus a vertical fin. For the combined shape to be physically possible, this fin must have positive thickness, that is to say the streamline $MIIB$, for example, in Fig. 24 must be deflected out of the plane $FLMI$ rather than into it. To ensure this it is necessary and sufficient that each of the two expansion axes lies on the same side of the plane of symmetry as the part of the wing which it is used to shape. Fig. 24(b) shows the axes correctly positioned; in Fig. 24(c) they are incorrectly positioned.

![Diagram](image-url)

Fig. 24. Use of Two Expansion Flows.

Fig. 25 Streamsurface in Axisymmetric Flow (Control Volume Shown Shaded).

4. The Use of Momentum Theory to Choose and Evaluate Lifting Surfaces. a) Introduction.

It has been shown in the preceding sections that there exists a class of axisymmetric flow fields (which are either known or may be computed relatively easily) from which chosen stream surfaces may be used as the basis of a rational design method to produce complete lifting configurations. In making practical use of this idea there are two main questions to be answered. Firstly, out of the infinity of flow fields which could be calculated, which are the ones most likely to yield practical, efficient shapes? Secondly, having made the choice of flow field, which of the infinity of stream surfaces contained within it will it be most appropriate to select? To some extent the choice is determined by geometrical constraints; that is the provision of a good structural shape with a convenient distribution of volume and a plausible aspect ratio. Good solutions to this geometrical problem may be found with the aid of some ingenuity. Section 5 of this paper gives examples of the processes involved.

The aerodynamic efficiency of such configurations is a rather less tangible problem. The methods described in this paper will automatically produce clean shapes without violent pressure gradients.
or surface shock waves, but although these are probably necessary conditions for efficiency they can hardly be supposed sufficient. One attempt to ensure aerodynamic efficiency would be to require that the load and volume distributions should be as closely as possible those recommended by linear theory, but in view of the Mach numbers involved this is unlikely to be more than a rough guide.

The approach adopted in this section is to use momentum theory to relate the lift and drag produced by a surface to certain properties of streamlines in the neighbouring flow. These properties may easily be calculated (for greatest convenience, simultaneously with the machine calculation of the basic flow field) and plotted so as to reveal from inspection the approximate lift and drag coefficients of any chosen streamsurface. Such plots will then assist the designer in producing the best compromise between aerodynamic and geometrical requirements.

We consider in what follows the application of this approach to the design of lower surfaces, results being presented for axisymmetric compression flows. As a further simplification we restrict ourselves to the case of a trailing edge lying in a plane normal to the free stream. An extension of this type of analysis to more general flow fields is published elsewhere [7]. An application of momentum theory to the special case of the cone flow wing has been given by Woods [15].

![Diagram of a 10° Cone at M = 1.0](image)

b) Momentum analysis. Consider a cylindrical co-ordinate system \((x,r,\psi)\) of which the axis is aligned parallel to a supersonic free stream. The flow field from which we shall choose a surface \(S\) is that behind some axisymmetric shock wave \(W\) (see Fig. 25) and we suppose the trailing edge of the surface to lie in some plane \(x = x_0\). The surface \(S\) may then be completely defined by drawing in planform the leading edge \(L\), which lies in \(W\). That portion of the shock bounded by \(L\) and \(TH\) will be unaltered when the stream surface is replaced by a solid surface. Consider a typical streamline \(PQ\) passing through this portion of the shock, and draw, on \(W\) and surrounding \(P\), a small closed curve \(C\) whose area in planview is \(dA_p\). When the space bounded by \(W, S,\) and \(x = x_0\) is considered as a control volume for the application of momentum theory it emerges that the contributions \(dL\) and \(dD\) made to the lift and pressure drag of \(S\) by the elementary streamtube through \(C\) are given by

\[
\begin{align*}
\text{d}L &= \left(\frac{1}{2} c_\infty v_\infty^2 \right) f_L(P) \text{d}A_p, \\
\text{d}D &= \left(\frac{1}{2} c_\infty v_\infty^2 \right) f_D(P) \text{d}A_p,
\end{align*}
\]

(4)

where

\[
\begin{align*}
f_L(P) &= (2 \tan \alpha) \left[ \frac{v_r}{v_\infty} \right], \\
f_D(P) &= \frac{2 \tan \alpha}{\cos \psi} \left[ 1 - \frac{v_x}{v_\infty} \frac{P - P_\infty}{\gamma P_\infty v_\infty^2} \right].
\end{align*}
\]

(5)
with the following notation: \( v_x, v_y \): velocity components at \( Q \); \( v_\infty \): free stream velocity; \( p \): static pressure at \( Q \); \( p_\infty \): free stream static pressure; \( \rho \): density at \( Q \); \( \rho_\infty \) free stream density; \( \sigma \): shock angle at \( P \).

It follows that if \( C_L \) and \( C_D \) are the coefficients of lift and pressure drag based on projected area, then \( C_L \) is the mean value over the chosen planform of \( f_L \) and \( C_D \) the mean value of \( f_D \).

If then we choose a flow field and a value of \( x_0 \), we may compute \( f_L \) and \( f_D \) for all points \( P \) in \( W \), and map onto the plan view of \( W \) contours of these functions. This has been done in Fig. 26 for the flow about a 10° cone at \( M = 4.0 \) (note that the conical property makes the results independent of \( x_0 \) in this case). It is at once evident that high performance will be associated with leading edges drawn close to \( ON \), producing sharply anhedralled surfaces.

A further example is shown in Fig. 27, where the generating body is a 15° cone-cylinder. The station is arbitrarily taken to be defined by the point at which the expansion from the shoulder meets the shock (see insert). Here we note the presence of a region of negative \( f_L \) near the apex of the shock, which would make a Type A design rather inefficient. The dashed line here shows the leading edge of the example given in Section 2 for which \( C_L = 0.076 \), and \( C_D = 0.0117 \).

An analysis of other flow fields, together with the effect of varying the trailing edge shape and notes on the maximum efficiencies achievable will be found in Ref. 7.

5. The Design of a Complete Configuration. In this section we indicate how the problem of designing a complete configuration can be approached. A flow diagram giving a rough indication of the procedures to be followed is presented as Fig. 28. At each of the decision-making stages, there will probably be more than one course of action available to remedy the defect, and which of these is taken will depend on the designer’s intuition.

Firstly, it is assumed that the \( Mach \) number and cruise lift coefficient have been prescribed, these being basic parameters determined by the project for which the vehicle is intended. The next step is to make an estimate of the percentage of the lift coefficient that is contributed by the wing lower surface. This estimate need only be very approximate in the first place and will be made
on a semi-empirical basis. Having thus approximately specified the value of the lower surface 

lift coefficient the next step is to design the lower surface. Since the lift coefficient is an average 

pressure coefficient we have a guide in the choice of axisymmetric body for the generation of the 

basic flow field. Further aspects to consider at this stage include the distribution of load over the 

fore and aft parts of the wing and the choice of wing planform. Some constraints upon wing planform 

will be imposed by consideration of the low speed regime etc., but within these constraints it 

will be possible to use the momentum methods previously described to determine those planforms 

giving the best lift to drag ratios for any basic axisymmetric flow field.

The next step is the design of the external geometry of the propulsion unit. It is assumed that 

the intake area and exit area are previously determined from the project specifications and engine 

calculations. We must now decide upon the longitudinal position and the shape of the intake (here, 

again, internal geometry considerations will be largely determining the external parameters). The 

external geometry of the propulsion unit will then be completed as described in Section 2 d.

If a streamwise upper surface is used (as in Fig. 20) the resulting configuration has a large base 

area. Part of this is occupied by engine exit area, but on either side of this it will be desirable to 

"carve away" volume from the rear end of the upper surface. To do this without causing shock
waves to appear on the surface we use the expansion flow techniques described in Section 3, and attempt to make the residual base area as shallow as possible over a large proportion of the span. The principal difficulties here are geometrical and the lifting efficiency of the upper surface is a secondary consideration. However, it will usually be found that the overall lift/drag ratio is improved by making the upper surface contribute to the lift.

It has been found that the use of two independent axisymmetric expansion flows, as described in Section 3e, greatly facilitates the achievement of a good base shape and a convenient volume distribution. Even so, it is not in general possible to realise the ideal base thickness everywhere, and in a region near the trailing edge it will be necessary to depart slightly from the calculated shape. As an example, the configuration shown in Figs. 19 and 20 has had its upper surface shaped in this way, and the resulting aircraft layout may be seen in Fig. 29.

Once a shape has been obtained which satisfies the basic design requirements it may then be subjected to further small empirical modifications, such as rounding the leading edges to meet aerodynamic heating requirements. It is reasonable to suppose that these modifications will not affect the main features of the flow, but this must, of course, be a matter for experimental checks. It may also be possible to calculate the effects of small perturbations of the basic flow field by making use of the method of linearised characteristics [16].

It remains for future project analysis to decide whether or not the present method can be used to produce a shape which meets all the requirements including cruise lift to drag ratio, off design characteristics, landing and take-off performance, distribution of stowage volume, satisfactory structure weight, and other aspects of overall aircraft design.

References


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