Theory of Ballasting Models (References 18 and 19)

To preserve dynamic similarity between a model and a corresponding airplane in a spin, the model must be ballasted so that it and the corresponding airplane describe geometrically similar helical paths. Thus, the helix angles of similar point paths must be equal, that is

$$\frac{\Omega_m l_m}{V_m} = \frac{\Omega_a l_a}{V_a}$$

the subscripts a and m referring to the airplane and model, respectively, l being any characteristic dimension.

For equilibrium and for similarity, the forces on model and airplane (centrifugal, gravitational, and aerodynamic) must equate to zero and, in addition, the summation of forces on model and airplane must form similar triangles. (It is assumed that there is no scale effect on the aerodynamic forces.) From this consideration are obtained the relations

$$\frac{W_a}{W_m} = \frac{\rho_a l_a^2 V_a^2}{\rho_m l_m^2 V_m^2} = \frac{M_a l_a \Omega_a^2}{M_m l_m \Omega_m^2}$$

(gravitational forces) (aerodynamic forces) (centrifugal forces)

Therefore $\Omega_a = \Omega_a \sqrt{N}$

$$\frac{W_a}{W_m} = \frac{M_a l_a \Omega_a^2}{M_m l_m \Omega_m^2}$$

and

$$V_a = V_m (N)^{-1/2}$$
where

$$W_a = \frac{\rho_a}{\rho_m} W_m(N)^{-3}$$

$N$ is the model scale (as $1/20$, $1/10$, etc.).

In addition to zero forces, the moments must also be in equilibrium in a spin (inertia moments due to rotation and the aerodynamic moments). For steady motion, these may be equated as

$$\frac{I_a \Omega_a^2}{I_m \Omega_m^2} = \frac{\rho_a l_a^3 V_a^2}{\rho_m l_m^3 V_m^2}$$

Then from the ratios given above, it is found that

$$I_a = \frac{\rho_a}{\rho_m} I_m(N)^{-5}$$

Thus, if the model is ballasted so that its weight is equivalent to $\frac{\rho_m}{\rho_a} W_a N^3$ and so that its moments of inertia about its three axes are equivalent to $\frac{\rho_m}{\rho_a} I_a N^5$, the model and airplane will have similar geometrical paths in the spin, and the rate of rotation and rate of descent in terms of full-scale values can be determined from the model test data by the formulas indicated above. The attitude in the spin and the turns required for recovery should be the same for model and airplane (assuming that there is no scale effect).

It is of interest to note that if the model is ballasted in this manner the Froude number $F = \frac{V}{\sqrt{\lambda g}}$ is the same for both model and airplane.
The Froude number is the law of mechanical similarity that must be satisfied when the inertia and gravitational forces are of primary importance, this is the case in spinning. For complete similarity, the Reynolds number should also be the same for model and airplane but it is impossible to have both the Reynolds number and the Froude number the same using air at atmospheric pressure as the test medium in the spin tunnel. The Reynolds number is equivalent to \( \frac{\mu_o V_L}{\rho} \); to satisfy both Reynolds number and Froude number the ratio of kinematic viscosity of the air at the altitude the airplane is to be spun and the kinematic viscosity of the air in the spin tunnel must be equivalent to the \( \frac{3}{2} \) power of the scale \( \frac{a}{m} = N^{-3/2} \).

For the present model, assuming the temperature of air in the tunnel the same as the air at airplane spin test altitude (5000 feet assumed), the air in the tunnel would have to be compressed to approximately 38 atmospheres to obtain the correct Reynolds number. It is felt that the Reynolds number is of secondary importance in spinning, however, inasmuch as the wing is usually completely stalled and thus changes in Reynolds number would be expected to have slight effect on the stall pattern or pressure distribution over the wing.
To minimize tunnel wall interference, you might select a model with an 8-ft. wing span, hence:

\[ \frac{b_m}{b} = \frac{8}{132} = \frac{1}{16.5} \]

From equation (2.0), the distributed weight ratio

\[ \frac{(mg)_m}{(mg)} = 1.82 \times \left(\frac{1}{4}\right)^2 \times \frac{1}{16.5} = \frac{1}{145.05} \]

or the model gross weight,

\[ W_m = (mg)_m \times b_m = \frac{1}{145.05} \times \frac{1}{16.5} \times 124,000 = 51.81 \text{ lbs} \]

As a check, assume a 1-g flight condition

Full Scale C-130A: 300 KTS = 507.15 ft/sec.

\[ G = \frac{1}{2} \rho V^2 = \frac{1}{2} (0.00127)(507.15)^2 = 163.32 \text{ lbs/ft}^2 \]

\[ S = 1600 \text{ ft}^2 \]

\[ C_L = \frac{124,000}{1600 \times 163.32} = 0.4745 \]

model: \[ S_m = 1600 \left(\frac{b_m}{b}\right)^2 = \frac{1600}{(16.5)^2} = 5.88 \text{ ft}^2 \]

\[ L_m = 0.4745 \times \frac{1}{2} (0.00231)(126.79)^2 \times 5.88 \]

\[ = 51.8 \text{ lbs} \quad \underline{CK.} \]