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A FINITE-DIFFERENCE SOLUTION FOR UNSTEADY WAVE INTERACTIONS
WITH AN APPLICATION TO THE EXPANSION TUBE

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A FINITE-DIFFERENCE SOLUTION FOR UNSTEADY WAVE INTERACTIONS
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Abstract

An analytical study of the unsteady wave interaction of a shock wave overtaking a previously initiated shock tube flow has been conducted for a real gas by solving a finite-difference representation of the one-dimensional Navier-Stokes equations. This analysis has been used to determine the effect on the available test time of opening the secondary diaphragm in the expansion-tube operating cycle prior to the arrival of the incident shock wave. Special emphasis is placed on determining expansion-tube test-time limitations resulting from a nonuniform entropy distribution occurring during the early test-gas flow due to the varying Mach number of the incident shock wave as it traverses the expansion fan caused by the preopening process. Time distributions of velocity, temperature, pressure, and entropy at various distances from the diaphragm are presented as a function of the time required to open the diaphragm. A comparison is made with available experimental data. The test time available for several different diaphragm opening times is presented for the various initial conditions considered. Although a particular solution is presented, the method of solution described is applicable to the general set of wave interaction problems.

Introduction

The operating cycle of the expansion tube\(^1\) requires that the incident shock wave in the intermediate chamber burst a thin, plastic diaphragm which separates the acceleration gas from the test gas. Experiments have shown that using the incident shock to burst the diaphragm can detract from the quality of the test gas flow as well as the efficiency of the operation. The nonideal operation can be traced to the test gas being processed by a nonplanar, reflected shock caused by a curved diaphragm and the transfer of energy from the flow to the accelerating diaphragm particles. Several methods of attacking this problem have been investigated. A device which removes the diaphragm from the flow prior to the arrival of the incident shock appears to have the most promise. Such a device is an electromagnetically opened diaphragm described by Jones.\(^2\) An electromagnetically opened diaphragm of different design was tested, with good results, by Weilmuenster,\(^3\) and used as the additional diaphragm required when the expansion tube is operated in the expansion tunnel mode. Experiment has shown that, at best, the system of Reference 2 can open a diaphragm of 7.62 cm diameter in 200 microseconds, while the system of Reference 3 is considerably slower.

Of concern when evaluating these systems for possible incorporation in a facility is—what influence they will have, for given running conditions, on the available test time. The parameter of interest is the time to open the diaphragm or, conversely, the time prior to the arrival of the incident shock at the diaphragm location, for which the diaphragm will not interfere with the passage of the shock. It is the purpose of this paper to investigate, by one-dimensional analysis, the effect of the preopening time \(\Delta t\) on the available test time. For these calculations, any effects of a two-dimensional diaphragm opening are neglected; that is, the diaphragm opening is instantaneous.

Analysis

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Analysis

A finite-difference solution of the full one-dimensional Navier-Stokes equations for a real, compressible gas was chosen as the most direct and computationally simple method of obtaining solutions to the given problem. There are (for example,4,5,6) many and varied approaches to the solution of the time-dependent Navier-Stokes equations. The method selected for this computation is the two-step Lax-Wendoff method as given by Thommen7 with slight modifications necessary to consider a real gas. This method is the easiest of the second-order accurate difference schemes to handle computationally and it handles shock waves without any additional computations such as shock fitting. Because of the higher order accuracy of the scheme, shock speeds are correct as opposed to first-order accurate schemes. Also Taylor et al.5 have shown that the accuracy of calculations of contact discontinuities and expansion waves are directly related to the accuracy of the differing method.

The equations, as presented here, are in the conservative or divergence-free form, and are written in a matrix form as:

\[ W_t = - F_x \]  

where

\[ W = \begin{bmatrix} \rho \\ \rho \mathbf{u} \\ \rho e \end{bmatrix} \]  

and

\[ F = \begin{bmatrix} \rho \mathbf{u} \\ \rho \mathbf{u} \mathbf{u} + \mathbf{p} \\ \rho \\ e + \frac{1}{2} \mathbf{u} \cdot \mathbf{u} \end{bmatrix} \]

along with the state equation \( \mathbf{p} = \mathbf{p}(e, \rho) \).

Determination of the pressure as a function of internal energy and density is accomplished through the use of a computer subroutine developed by Tannehill and Mohling.6 This subroutine uses curve fits of data generated by the NASA-ARC ENAS computer program for the thermodynamic properties of equilibrium air. In addition, the subroutine calculates \( T = \mathbf{T}(e, \rho) \) and \( a = a(e, \rho) \). Entropy distributions were obtained from the pressure and density using computer subroutine SAVF developed by Miller9 for equilibrium air.

The equations are solved in the nondimensional form where

\[ U = \frac{\mathbf{u}}{U_0}, \quad x = \frac{x}{x_0}, \quad t = \frac{t U_0}{x_0} \]

\[ P = \frac{\mathbf{p}}{P_0}, \quad e = \frac{\mathbf{e}}{U_0 T_0}, \quad T = \frac{\mathbf{T}}{T_0}, \quad P = \frac{\mathbf{p}}{P_0 T_0} \]
Weilmuenster: Unsteady Wave Interactions

which gives

\[
\begin{bmatrix}
\rho \\
pU \\
\rho e + \frac{U_0^2}{2C_v} + U^2
\end{bmatrix} = \begin{bmatrix}
pU \\
pU^2 + \frac{R}{U_0^2} + \rho \frac{U_0^2}{C_v} + \rho U_0 \\
U_0 \frac{R}{C_v} + \rho \frac{U_0^2}{C_v} + \frac{U_0^2}{2C_v} + \rho U_0
\end{bmatrix} x
\]

The computational grid is as follows:

\[
t + \Delta t, x, x \pm \Delta x
\]

The difference equations used for the solution of Equations (4) are:

\[
W(t + \Delta t, x) = W(t, x) + \frac{1}{2} (W(t, x) + W(t, x + \Delta x)) + \frac{\lambda}{2} (P(t, x) - P(t, x + \Delta x))
\]

and

\[
W(t + \Delta t, x) = W(t, x) - \lambda (P(t + \Delta t, x + \Delta x) - P(t + \Delta t, x))
\]

where \(\lambda\) is the Courant-Friedrichs-Lewy stability criterion

\[
\lambda = \frac{\Delta t}{\Delta x} \leq \left| \frac{U + e}{\max} \right|
\]

To begin a calculation, it is only necessary to specify the linear distribution of \(\rho, U, e,\) and \(P\). Thereafter, only the first and last point in the computational grid need be specified. The solution is advanced in time by determining the minimum \(\lambda\) in the computational field (thus determining \(\Delta t\) since \(\Delta x\) is fixed) and operating on the appropriate grid points with Equations (5) and (6).

A schematic of the wave system resulting from the interaction of the incident shock wave and the flow initiated by the preopened diaphragm is shown on Figure 1. The thermodynamic state of Regions 1, 2, 10, 20, and the test gas 5 are identical to Trimmel's for an ideal diaphragm opening as shown in the upper left-hand corner of Figure 1. As is seen in Figure 1, the incident shock, \(S_1\), as it traverses the expansion, \(E_1\), initiated by the preopening, creates an entropy distribution due to varying shock Mach number. The trailing edge of the nonisentropic region, interface \(I_5\), is the streamline whose origin is the intersection of \(S_1\) and the leading edge of \(E_1\). Before there is any available test-time, \(I_5\) must exit the expansion fan, \(E_2\), created by the intersection of \(S_1\) and the secondary shock \(S_2\). The essence of the problem then is to determine the intersection of \(I_5\) and the trailing edge of \(E_2\).

To determine the origin of Region 5, the computational grid was initialized by specifying the position of the incident shock and the secondary diaphragm as well as the then allow computational origin of

Results

Three items of interest are shown: corresponding shock are internal at occur at (see Ref. 2 of the work throughout the theoretical interface \(S_2\) and the information which plotted in in the later also a parameter \(S_1\) and \(S_2\) represents a low value represents \(U_1\) and \(a\) there is a Region 5 for.

The diaphragm is at a given value the origin of

The parameters for constant respectively 200 microseconds the case.
well as the thermodynamic state of Regions 1, 2, and 10. The computation was then allowed to proceed in time until the properties of Region 5 appeared in the computational grid. 13 and the trailing edge of Region 5 were plotted and the origin of Region 5 was determined by the intersection of these two curves.

Results

Three cases, see Table 1, were run which are indicative of the varied operation of the expansion tube. Case 1 is representative of the operational mode employed when the expansion tube is used in conjunction with a nozzle, that is, an expansion tunnel, for testing in high Reynolds number flow. Case 2 is representative of the highly expanded mode of operation used for earth-orbital studies. Case 3, if highly expanded, would be used for planetary entry studies, however, since relaxation processes in E1 being studied, an elevated value of T5 was used.

Axial distributions of pressure, temperature, velocity, and entropy for a Δt = 82 microseconds at 4.4222 milliseconds after the diaphragm was opened are shown in Figure 2 for Case 1. The values at the extreme right in each plot correspond to the location of 𝑘. The oscillations in parameters behind the shock are the result of the manner in which the differencing method handles an internal shock. The oscillations in pressure, temperature, and velocity which occur at the trailing edge of E2 are the result of the differencing scheme (see Ref. 5). However, these oscillations in no way affect the overall accuracy of the solution. This is best pointed out by the entropy which remains constant throughout the disturbance. The dashed lines in each of the four plots represent the theoretical value of the respective parameters in Region 5. The location of the interfaces I1, I2, and I3 are shown on the entropy plot. The position of S1 and the trailing edge of E2 are shown on the velocity plot. Comparable information for Case 2 is shown on Figure 3 for a Δt = 100 microseconds at 3.1711 milliseconds after the diaphragm opening. The origin of Region 5 is plotted in Figure 4 for both Case 1 and Case 2 as a function of Δt. This origin is dependent upon 𝑈_{S1}. Since the farther downstream E2 originates, the later I3 will intersect the trailing edge of E2; the origin of E2 is also a parameter of interest. Since the origin of E2 is the intersection of S1 and S2', the governing parameter is P1/P10 which dictates 𝑈_{S2}'. Case 1 represents the least desirable combination of the parameters 𝑈_{S1} and P1/P10, a low value of 𝑈_{S1} and a high-pressure ratio across the diaphragm. Case 2 represents the most desirable combination of the parameters, a high value of 𝑈_{S1} and a low-pressure ratio across the diaphragm. As Figure 4 indicates, there is a considerable difference in the downstream position of the origin of Region 5 for the two cases.

The amount of test time lost, ΔTloss, due to the preopening of the diaphragm is shown for both Cases 1 and 2 in Figure 5 as a function of Δt. For a given value of Δt, ΔTloss is defined as the ideal test time available at the origin of Region 5.

The percentage of the ideal test time available downstream of the diaphragm for constant values of Δt are shown on Figures 6 and 7, respectively. Since reasonable diaphragm opening times are on the order of 200 microseconds, the test time ratio available 12 meters downstream of the diaphragm would be 92% in the most favorable case and 38% in the least favorable case.
Case 3 was run for comparison with experimental results obtained in the Langley Pilot Model Expansion Tube using an electromagnetically opened diaphragm similar to that described in Reference 2. Theoretical and experimental values of the total pressure, \( P_t \), and pressure approximately 10 m downstream of the diaphragm are plotted in Figures 8 and 9, respectively, as a function of time after diaphragm opening. Due to an uncertainty in exactly when the diaphragm opens and viscous effects, the wave arrival times do not exactly match. However, the theoretical calculations predict the correct width of the aggregate interfaces \( I_1 \), \( I_2 \), and \( I_3 \) as well as the correct relative positions of \( I_2 \). The discrepancy in the experimental level of the total pressure in Region 5 could have been anticipated from previous experiments, both with and without pre-opened diaphragms. Jones and Moore\(^1\) show that measurements of total pressure on the order of one-half the theoretical value are not infrequent; and they attributed the difference to several factors, the most dominant of which is the shock attenuation. The difference in total pressure levels in Region 20 are to be anticipated since the Region 10 gas used in the theoretical calculation was air, while that used in the experiment was helium. The theoretical and experimental static pressure records are shown on Figure 9 for Case 3. As reported by Jones and Moore\(^1\), the difference in the static pressure levels for the two cases in Regions 20 and 5 is primarily the effect of shock attenuation.

Summary

A finite-difference computer program which solves the one-dimensional Navier-Stokes equations for a real gas has been used to determine the effect on the test time of preopening the secondary diaphragm in the expansion tube operating cycle. It was found that even under the constraints of available opening devices, the least desirable modes of operation still allowed a reasonable test time. A comparison of theoretical and experimental values showed that the one-dimensional assumptions used in this analysis gave a reasonably accurate description of the flow process. It must be left to experiment, however, to determine if the penalties paid in test time are justified in overall improvement in the quality of the test gas.

References


Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>a</td>
<td>Sound speed</td>
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<tr>
<td>CV</td>
<td>Specific heat at constant volume</td>
</tr>
<tr>
<td>e</td>
<td>Internal energy</td>
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<tr>
<td>F(W)</td>
<td>Vector valued function</td>
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<tr>
<td>P</td>
<td>Pressure</td>
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<tr>
<td>R</td>
<td>Gas constant</td>
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<td>S</td>
<td>Entropy</td>
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<td>T</td>
<td>Temperature</td>
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<tr>
<td>t</td>
<td>Time</td>
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<tr>
<td>U</td>
<td>Velocity</td>
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<tr>
<td>W</td>
<td>Column vector, dependent variable</td>
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<table>
<thead>
<tr>
<th>Superscripts</th>
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<tr>
<td>X</td>
<td>Δt</td>
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\[
\frac{\Delta t}{\Delta t} = \frac{p_{\text{opening}} - p_0}{T}
\]

\[
\frac{\Delta t}{\text{loss}} = \text{loss of test time caused by pre-opening expansion wave}
\]

\[
\frac{\Delta t}{\text{loss}} = \text{entraping air}
\]
Figure 1. Comparison of wave diagrams for an ideal and preopened diaphragm.
Figure 2. Case 1, time after diaphragm opening 4.4522 msec, \( t_{\text{pot}} = 82 \mu\text{sec} \).
Figure 3. Case 2, time after diaphragm opening 3.1711 msec, $t_{pot} = 100$ μsec.
Figure 4. Origin of Region 5 versus time after diaphragm opening.

Figure 5. Test time lost due to time required to open diaphragm.
Figure 6. Percentage of ideal test time available downstream of diaphragm for constant $\Delta t_{pot}$, Case 1.

CASE 1
IDEAL $\Delta_t_{test} = 58.5$ $\mu$sec/m

CASE 2
IDEAL $\Delta_t_{test} = 33$ $\mu$sec/m

Figure 7. Percentage of ideal test time available downstream of diaphragm for constant $\Delta t_{pot}$, Case 2.
Figure 8. Comparison of theoretical and experimental values of the total pressure for Case 3, 10.466 m downstream of the diaphragm.

Figure 9. Comparison of theoretical and experimental values of the static pressure for Case 3, 10.286 m downstream of the diaphragm.